Last Name:
Name:
Instructor:

Math 150
Group Final (Spring 2006)
Version A

You are not allowed to use notes, books, calculators, personal stereos or cell phones. You have exactly 2 hours.
This is a multiple-choice exam. You will use only the provided blank work packet for the calculations. At the end of the exam you will turn in the scantron, the question packet and the work packet with your name on each item. Any missing item may result in disciplinary action and failure in the course. Make sure that your red ID is on both sides of the scantron.

1.

$$\lim_{x \to 4-} \frac{\left|x^2 - 3x - 4\right|}{2x - 8}$$

is

a) $+\infty$ *b*) $-\infty$ *c*) $\frac{5}{2}$ *d*) $-\frac{5}{2}$ *e*) 0

2.

$$\lim_{x \to \pi/2+} \frac{x^2 + 1}{\cos(x)}$$

is

a) $+\infty$ *b*) $-\infty$ *c*) $\frac{\pi^2}{4} + 1$ *d*) 0 *e*) nonexistent

3.

$$\lim_{x \to 0} \frac{\arctan(x) - x}{x^3}$$

is

a) 0 *b*) $+\infty$ *c*) $-\frac{1}{3}$ *d*) $\frac{1}{3}$ *e*) $-\frac{1}{6}$

4.

$$\lim_{x \to +\infty} \frac{\ln(x)}{x^{2/3}}$$

is

a) $+\infty$ *b*) 0 *c*) 1 *d*) -1 *e*) nonexistent

5.

$$\lim_{x \to +\infty} \left(1 + x^2\right)^{1/x}$$

is

a) e *b*) e^2 *c*) $+\infty$ *d*) 0 *e*) 1

6. If $f(x) = \dfrac{3x+1}{x^2+9}$ then $f'(x)$ is

a) $\dfrac{-3x^2-2x+27}{x^2+9}$ *b)* $\dfrac{3x^2+2x-27}{(x^2+9)^2}$ *c)* $\dfrac{3x^2+2x-27}{x^2+9}$

d) $\dfrac{-3x^2-2x+27}{(x^2+9)^2}$ *e)* $\dfrac{9x^2+2x+27}{(x^2+9)^2}$

7. If $f(t) = \cos^2(\pi t)$ then $f'(1/3)$ is

a) $\dfrac{\sqrt{3}}{2}$ *b)* $\dfrac{\sqrt{3}}{2}\pi$ *c)* $-\dfrac{\sqrt{3}}{2}\pi$ *d)* $-\dfrac{\sqrt{3}}{2}$ *e)* $\sqrt{3}\pi$

8. If $f(x) = \dfrac{1}{\sqrt{x^4+1}}$ then $f'(x)$ is

a) $\dfrac{4x^3}{(x^4+1)^{3/2}}$ *b)* $\dfrac{2x^3}{(x^4+1)^{3/2}}$ *c)* $-\dfrac{2x^3}{\sqrt{x^4+1}}$

d) $\dfrac{2x^3}{\sqrt{x^4+1}}$ *e)* $-\dfrac{2x^3}{(x^4+1)^{3/2}}$

9.

$$\frac{d}{dx}10^{-x^2}$$

is

a) $-2x10^{-x^2-1}$ *b)* $-2x\ln(10)\,10^{-x^2}$ *c)* $-2x10^{-x^2}$

d) $-2x\ln(10)\,10^{-x}$ *e)* $-2x\ln(10)\,10^{-x^2-1}$

10. If $f(x) = \arcsin(x/4)$, then $f'\left(\sqrt{7}\right)$ is

a) $-\dfrac{4}{3}$ *b)* $\dfrac{4}{3}$ *c)* $\dfrac{1}{3}$ *d)* $\dfrac{1}{9}$ *e)* $\dfrac{4}{9}$

11. The equation of the tangent line to the graph of $f(x) = \dfrac{x}{x^2+4}$ at $(-1, f(-1))$ is

a) $y = \dfrac{3}{25}x - \dfrac{8}{25}$ *b)* $y = -\dfrac{1}{5}x + \dfrac{3}{5}$ *c)* $y = \dfrac{1}{5}x - \dfrac{3}{25}$

d) $y = \dfrac{3}{25}x - \dfrac{2}{25}$ *e)* $y = -\dfrac{1}{5}x - \dfrac{3}{5}$

12. Let $f(x) = \tan(x)$. The approximation to $\tan\left(\dfrac{3\pi}{4} - 0.1\right)$ via the linear approximation to f based at $3\pi/4$ (the linearization of f at $3\pi/4$) is

a) -0.8 *b)* 1.2 *c)* -1.2 *d)* -1.1 *e)* 1.1

13. If the position of a particle moving along the x-axis is $f(t) = \cos\left(10t - \frac{\pi}{6}\right)$ at time t, then the acceleration at t is

a) $-10\sin\left(10t - \frac{\pi}{6}\right)$ *b)* $10\sin\left(10t - \frac{\pi}{6}\right)$
c) $100\cos\left(10t - \frac{\pi}{6}\right)$ *d)* $-100\cos\left(10t - \frac{\pi}{6}\right)$ *e)* $10\cos\left(10t - \frac{\pi}{6}\right)$

14. A plane flying horizontally at an altitude of 2 miles and a speed of 400 miles per hour passes directly over a radar station. Find the rate at which the distance from the plane to the station is increasing when it is 3 miles away from the station.

a) $\frac{400\sqrt{5}}{9}$ *b)* $\frac{1200}{\sqrt{5}}$ *c)* $\frac{400\sqrt{5}}{3}$ *d)* $\frac{2000}{3}$ *e)* $\frac{400\sqrt{3}}{\sqrt{5}}$

15. Let $f(x) = x^4 e^{-x}$. The absolute maximum of f on the interval $[0, +\infty)$ is

a) $\frac{4^3}{e^3}$ *b)* $\frac{16}{e^2}$ *c)* $\frac{81}{e^3}$ *d)* $\frac{256}{e^4}$ *e)* none of these

16. Let $f(x) = \frac{1}{12}x^4 + \frac{1}{6}x^3 - 6x^2$ The graph of f is concave down on

a) $(-3, 4)$ *b)* $(3, 4)$ *c)* $(-4, 3)$
d) $(-\infty, -4)$ and $(3, +\infty)$ *e)* $(-\infty, -3)$ and $(4, +\infty)$

17. Let

$$f(x) = -2x + \frac{1}{2}x^2 + \frac{1}{3}x^3.$$

The function f attains its absolute maximum on the interval $[-3, 2]$ at

a) -3 *b)* 2 *c)* 1 *d)* -1 *e)* -2

18.

$$\int_{-1}^{0} \frac{d}{dx}\left(\sqrt{\frac{4-x^2}{4+x^2}}\right) dx$$

is

a) $\frac{\sqrt{15}-1}{5}$ *b)* $1 - \frac{\sqrt{15}}{5}$ *c)* $\frac{-3}{5}$ *d)* $\frac{3}{5}$ *e)* $\frac{1-\sqrt{3}}{3}$

19.

$$\frac{d}{dx}\int_{x}^{\pi} e^{\sin^2(t)} dt$$

is

a) $2\, e^{\sin^2(x)} \sin(x)\cos(x)$ *b)* $-2e^{\sin^2(x)} \sin(x)\cos(x)$ *c)* $e^{\sin^2(x)}$
d) $-\, e^{\sin^2(x)}$ *e)* $e^{2\sin(x)\cos(x)}$

20.

$$\int_{\sqrt{\pi/3}}^{\sqrt{\pi/2}} x \sin\left(x^2\right) dx$$

is

$a)\ -\frac{1}{4}$ $\quad b)\ \frac{1}{4}$ $\quad c)\ \frac{\sqrt{3}}{4}$ $\quad d)\ -\frac{\sqrt{3}}{4}$ $\quad e)\ -\frac{\sqrt{3}}{2}$

21.

$$\int_{\sqrt{4+e}}^{\sqrt{4+2e}} \frac{x}{4-x^2} dx$$

is

$a)\ -\frac{1}{2}$ $\quad b)\ \frac{1}{2}$ $\quad c)\ -\frac{1}{2}\ln(2)$ $\quad d)\ \frac{1}{2}\ln(2)$ $\quad e)$ none of these

22.

$$\int_0^4 \frac{x}{\sqrt{x^2+9}} dx$$

is

$a)\ -1$ $\quad b)\ \frac{1}{2}$ $\quad c)\ 2$ $\quad d)\ 1$ $\quad e)\ \ln(2)$

23. The area of the region between the graph of $f(x) = x^2 - x - 2$, the interval $[0, 3]$, the line $x = 0$ and the line $x = 3$ is

$a)\ \frac{3}{2}$ $\quad b)\ \frac{2}{3}$ $\quad c)\ \frac{21}{3}$ $\quad d)\ \frac{31}{6}$ $\quad e)\ \frac{10}{3}$

24. Assume that the velocity of an object moving along the x-axis is $\cos(4t)$ at time t, and its position at $t = \pi/8$ is 2. The position of the object at $t = 3\pi/8$ is

$a)\ \frac{7}{2}$ $\quad b)\ \frac{3}{2}$ $\quad c)\ \frac{1}{2}$ $\quad d)\ 3$ $\quad e)\ \frac{11}{4}$

25. Assume that

$$\frac{dy}{dt} = te^{-t^2} \text{ and } y\left(\sqrt{\ln(2)}\right) = 4.$$

Then $y(t)$ is

$a)\ 3 - e^{-t^2}$ $\quad b)\ 3 - \frac{1}{2}e^{-t^2}$ $\quad c)\ \frac{17}{4} - \frac{1}{2}e^{-t^2}$

$d)\ \frac{17}{4} + \frac{1}{2}e^{-t^2}$ $\quad e)\ \frac{1}{2} - \frac{1}{2}e^{-t^2}$

Solutions

1.

$$\lim_{x\to 4-}\frac{\left|x^2-3x-4\right|}{2x-8}=\lim_{x\to 4-}\frac{\left|(x+1)(x-4)\right|}{2(x-4)}=\lim_{x\to 4-}\frac{(x+1)(4-x)}{2(x-4)}=\lim_{x\to 4-}\frac{-(x+1)}{2}=-\frac{5}{2}$$

The answer is **d.**

2. We have

$$\lim_{x\to \pi/2}\left(x^2+1\right)=\frac{\pi^2}{4}+1>0,$$

and

$$\lim_{x\to \pi/2-}\frac{1}{\cos(x)}=-\infty \text{ since } \cos(x)<0 \text{ if } \pi/2<x<\frac{3\pi}{2} \text{ and } \lim_{x\to \pi/2}\cos(x)=\cos(0)=0.$$

Therefore,

$$\lim_{x\to \pi/2+}\frac{x^2+1}{\cos(x)}=\lim_{x\to \pi/2-}\left(\left(x^2+1\right)\left(\frac{1}{\cos(x)}\right)\right)=-\infty.$$

The answer is **b.**

3. By L'Hôpital's rule,

$$\lim_{x\to 0}\frac{\arctan(x)-x}{x^3}=\lim_{x\to 0}\frac{\dfrac{1}{x^2+1}-1}{3x^2}=\lim_{x\to 0}-\frac{1}{3\left(x^2+1\right)}=-\frac{1}{3}.$$

The answer is **c.**

4. By L'Hôpital's rule,

$$\lim_{x\to +\infty}\frac{\ln(x)}{x^{2/3}}=\lim_{x\to \infty}\frac{\dfrac{1}{x}}{\dfrac{2}{3}x^{-1/3}}=\lim_{x\to \infty}\frac{3}{2x^{2/3}}=0.$$

The answer is **b.**

5. We have

$$\begin{aligned}\lim_{x\to +\infty}\left(1+x^2\right)^{1/x}=\lim_{x\to +\infty}\exp\left(\frac{1}{x}\ln\left(1+x^2\right)\right) &= \exp\left(\lim_{x\to +\infty}\frac{1}{x}\ln\left(1+x^2\right)\right)\\ &= \exp\left(\lim_{x\to \infty}\frac{\dfrac{2x}{1+x^2}}{1}\right)\\ &= \exp(0)=1.\end{aligned}$$

The answer is **e**.

6.

$$\frac{d}{dx}\left(\frac{3x+1}{x^2+9}\right)=\frac{3\left(x^2+9\right)-(3x+1)(2x)}{\left(x^2+9\right)^2}=\frac{-3x^2-2x+27}{\left(x^2+9\right)^2}$$

The answer is **d.**

7.

$$f'(t)=\frac{d}{dt}\cos^2(\pi t)=-2\pi\cos(\pi t)\sin(\pi t)$$

Therefore,

$$f'\left(\frac{1}{3}\right)=-2\pi\cos\left(\frac{\pi}{3}\right)\sin\left(\frac{\pi}{3}\right)=-2\pi\left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right)=-\frac{\sqrt{3}}{2}\pi$$

The answer is **c.**

8.

$$f'(x)=\frac{d}{dx}\frac{1}{\sqrt{x^4+1}}=\frac{d}{dx}\left(x^4+1\right)^{-1/2}=-\frac{1}{2}\left(x^4+1\right)^{-3/2}\left(4x^3\right)=-\frac{2x^3}{\left(x^4+1\right)^{3/2}}$$

The answer is **e.**

9.

$$\frac{d}{dx}10^{-x^2}=-2x\ln(10)\,10^{-x^2}.$$

The answer is **b.**

10. We have

$$f'(x)=\frac{d}{dx}\arcsin\left(\frac{x}{4}\right)=\frac{1}{\sqrt{1-\frac{x^2}{16}}}\left(\frac{1}{4}\right)=\frac{1}{\sqrt{16-x^2}}.$$

Therefore,

$$f'\left(\sqrt{7}\right)=\frac{1}{\sqrt{16-7}}=\frac{1}{\sqrt{9}}=\frac{1}{3}.$$

The answer is **c.**

11. We have

$$f'(x)=\frac{d}{dx}\left(\frac{x}{x^2+4}\right)=\frac{x^2+4-2x^2}{\left(x^2+4\right)^2}=\frac{4-x^2}{\left(x^2+4\right)^2}..$$

Therefore,

$$f(-1)=\frac{-1}{5}\text{and }f'(-1)=\frac{3}{25}$$

Thus,

$$y=-\frac{1}{5}+\frac{3}{25}(x+1)=\frac{3}{25}x-\frac{2}{25}$$

The answer is **d.**

12. We have

$$f'(x) = \frac{d}{dx}\tan(x) = \frac{1}{\cos^2(x)}.$$

Therefore,

$$f\left(\frac{3\pi}{4}\right) = \tan\left(\frac{3\pi}{4}\right) = -1, \text{ and } f'\left(\frac{3\pi}{4}\right) = \frac{1}{\left(\frac{1}{\sqrt{2}}\right)^2} = 2.$$

Thus,

$$L(x) = f\left(\frac{3\pi}{4}\right) + f'\left(\frac{3\pi}{4}\right)\left(x - \frac{3\pi}{4}\right) = -1 + 2\left(x - \frac{3\pi}{4}\right).$$

Therefore,

$$\tan\left(\frac{3\pi}{4} - 0.1\right) \cong L\left(\frac{3\pi}{4} - 0.1\right) = -1 + 2(-0.1) = -1.2$$

The answer is **c.**

13. We have

$$v(t) = \frac{d}{dt}\cos\left(10t - \frac{\pi}{6}\right) = -10\sin\left(10t - \frac{\pi}{6}\right),$$

and

$$a(t) = \frac{dv}{dt} = \frac{d}{dt}\left(-10\sin\left(10t - \frac{\pi}{6}\right)\right) = -100\cos\left(10t - \frac{\pi}{6}\right).$$

The answer is **d.**

14.
Let $s(t)$ be the distance of the plane from the radar station and let $x(t)$ be the horizontal coordinate that is 0 at the instant the plane is directly above the station. We have

$$s^2(t) = x^2(t) + 4$$

so that

$$2s(t)\frac{ds}{dt} = 2x(t)\frac{dx}{dt} \Rightarrow \frac{ds}{dt} = 400\frac{x(t)}{s(t)}.$$

At the instant $s = 3$, we have

$$x = \sqrt{9-4} = \sqrt{5}.$$

Therefore,

$$\frac{ds}{dt} = 400\left(\frac{\sqrt{5}}{3}\right) = \frac{400\sqrt{5}}{3} \text{ (miles per hour).}$$

The answer is **c.**

15. We have

$$f'(x) = 4x^3e^{-x} - x^4e^{-x} = x^3e^{-x}(4-x) = 0$$

if $x = 0$ or $x - 4$. Since $f'(x) > 0$ if $0 < x < 4$ and $f'(x) < 0$ if $x > 4$, f is increasing on $[0, 4]$ and decreasing on $[4, +\infty)$. Therefore, the absolute maximum of f on $[0, +\infty)$ is

$$f(4) = 4^4e^{-4} = \frac{256}{e^4}$$

The answer is **d.**

16. We have

$$f'(x) = \frac{d}{dx}\left(\frac{1}{12}x^4 + \frac{1}{6}x^3 - 6x^2\right) = \frac{1}{3}x^3 + \frac{1}{2}x^2 - 12x$$

and

$$f''(x) = \frac{d}{dx}\left(\frac{1}{3}x^3 + \frac{1}{2}x^2 - 12x\right) = x^2 + x - 12 = (x+4)(x-3)$$

Therefore, $f''(x) < 0$ if $x \in (-4,3)$ Thus, the graph of f is concave down on the intervals $(-4,3)$. The answer is **c.**

17. We have

$$f'(x) = -2 + x + x^2 = 0 \Leftrightarrow x = -2 \text{ or } x = 1.$$

Since

$$f(-2) = \frac{10}{3} \cong 3.3,\ f(1) = -\frac{7}{6} \cong -1.16,\ f(-3) = \frac{3}{2} = 1.5,\ f(2) = \frac{2}{3} \cong 0.67\,,$$

the absolute maximum of f on $[-3,2]$ is at -2..
The answer is **e.**

18.

$$\int_{-1}^{0} \frac{d}{dx}\left(\sqrt{\frac{4-x^2}{4+x^2}}\right)dx = \sqrt{\frac{4-x^2}{4+x^2}}\Bigg|_{-1}^{0} = 1 - \sqrt{\frac{3}{5}} = 1 - \frac{\sqrt{15}}{5}.$$

The answer is **b.**

19.

$$\frac{d}{dx}\int_{x}^{\pi} e^{\sin^2(t)}dt = \frac{d}{dx}\left(-\int_{\pi}^{x} e^{\sin^2(t)}dt\right) = -e^{\sin^2(x)}$$

The answer is **d.**

20. We set $u = x^2$ so that $du = 2xdx$. Thus,

$$\int_{\sqrt{\pi/3}}^{\sqrt{\pi/2}} x\sin\left(x^2\right)dx = \frac{1}{2}\int_{\pi/3}^{\pi/2} \sin(u)\,du = \frac{1}{2}\left(-\cos(u)|_{\pi/3}^{\pi/2}\right) = \frac{1}{2}\left(\frac{1}{2}\right) = \frac{1}{4}.$$

The answer is **b.**

21. We set $u = 4 - x^2$ so that $du = -2xdx$. Thus,

$$\begin{aligned}\int_{\sqrt{4+e}}^{\sqrt{4+2e}} \frac{x}{4-x^2}dx = -\frac{1}{2}\int_{-e}^{-2e} \frac{1}{u}du = -\frac{1}{2}\left(\ln(|u|)|_{-e}^{-2e}\right) &= -\frac{1}{2}\left(\ln(2e) - \ln(e)\right) \\ &= -\frac{1}{2}\left(\ln(2) + 1 - 1\right) = -\frac{1}{2}\ln(2)\end{aligned}$$

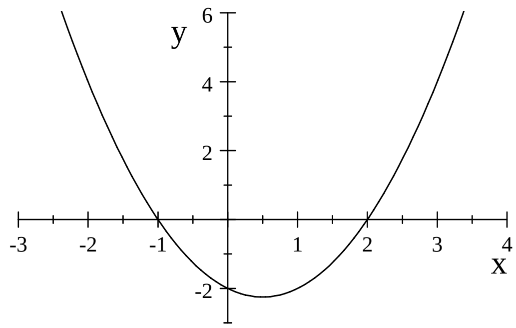
The answer is **c**.

22. We set $u = x^2 + 9$ so that $du = 2dx$. Thus,

$$\int_{0}^{4} \frac{x}{\sqrt{x^2+9}}dx = \frac{1}{2}\int_{9}^{25} u^{-1/2}du = \frac{1}{2}\left(2u^{1/2}\Big|_{9}^{25}\right) = 5 - 3 = 2.$$

The answer is **c.**

23.



We have

$$x^2 - x - 2 = 0 \Leftrightarrow x = -1 \text{ or } x = 2.$$

The required area is

$$\begin{aligned} -\int_0^2 \left(x^2 - x - 2\right) dx + \int_2^3 \left(x^2 - x - 2\right) dx &= -\left(\frac{1}{3}x^3 - \frac{1}{2}x^2 - 2x\Big|_0^2\right) + \left(\frac{1}{3}x^3 - \frac{1}{2}x^2 - 2x\Big|_2^3\right) \\ &= \frac{10}{3} + \frac{11}{6} = \frac{31}{6} \end{aligned}$$

The answer is **d.**

24.
The position of the object at $t = 3\pi/8$ is

$$\begin{aligned} 2 + \int_{\pi/8}^{3\pi/8} \cos(4t)\, dt = 2 + \left(\frac{1}{4}\sin(4t)\Big|_{\pi/8}^{3\pi/8}\right) &= 2 + \frac{1}{4}\left(\sin\left(\frac{3\pi}{2}\right) - \sin\left(\frac{\pi}{2}\right)\right) \\ &= 2 + \frac{1}{4}(-2) = \frac{3}{2} \end{aligned}$$

The answer is **b.**

25.

$$\begin{aligned} y(t) = 4 + \int_{\sqrt{\ln(2)}}^{t} ue^{-u^2} du = 4 - \frac{1}{2}\int_{-\ln(2)}^{-t^2} e^v dv &= 4 - \frac{1}{2}\left(e^{-t^2} - \frac{1}{2}\right) \\ &= \frac{17}{4} - \frac{1}{2}e^{-t^2} \end{aligned}$$

The answer is **c.**