Revised on 11/14/2005

Math 150 Fall 2005
Sample Problems for the Multiple Choice Segment of the Final Exam

1. If $f(x) = \sin^2(2x)$, then $f'(\pi/8)$ is

$a)\ 0 \qquad b)\ -2 \qquad c)\ 2 \qquad d)\ 1 \qquad e)\ \dfrac{\pi}{2}$

2.

$$\lim_{x \to +\infty} \frac{4x^3 - 5x}{x^3 + 6x - 6}$$

is

$a)\ 0 \qquad b)\ +\infty \qquad c)\ 4 \qquad d)\ \dfrac{2}{3} \qquad e)$ nonexistent

3. The equation of the tangent line to the graph of $y = 2x^3 - 4x$ at the point $(2, 8)$ is

$a)\ 13x-8 \qquad b)\ 8-20x \qquad c)\ 8y = 20(x-1) \qquad d)\ y = 2x+8 \qquad e)\ y = 20x-32$

4. If $y = \sin(3x)\cos(2x)$, then $\dfrac{dy}{dx} =$

$a)\ \cos(3x) - \sin(2x)$
$b)\ \sin(3x)\sin(2x) + \cos(3x)\cos(2x)$
$c)\ 2\sin(3x)\sin(2x) + 3\cos(3x)\cos(2x)$
$d)\ 3\sin(3x)\sin(2x) + 2\cos(3x)\cos(2x)$
$e)\ 3\cos(3x)\cos(2x) - 2\sin(3x)\sin(2x)$

5. If $f(x) = (x-1)^2 \sin(x)$, then $f'(0) =$

$a)\ -2 \qquad b)\ -1 \qquad c)\ 0 \qquad d)\ 1 \qquad e)\ 2$

6. If the position of a particle moving along the x-axis is $f(t) = t^3 - t^2 + 4t + 6$ at time t, then the acceleration at $t = 2$ is

$a)\ 2 \qquad b)\ 12 \qquad c)\ 6 \qquad d)\ 10 \qquad e)\ 0$

7. For what value of x does the function $f(x) = x^3 - 3x$ has a local (relative) maximum?

$a)\ -3 \qquad b)\ 1 \qquad c)\ -1 \qquad d)\ 2 \qquad e)\ 3$

8.

$$\frac{d}{dx}\int_2^x \cos\left(4\pi^2 u^2\right) du$$

is

$a)\ 0$ $b)\ \frac{1}{4\pi^2}\sin\left(4\pi^2 x^2\right)$ $c)\ -\frac{1}{4\pi^2}\sin\left(4\pi^2 x^2\right)$
$d)\ \cos\left(4\pi^2 x^2\right)$ $e)\ \cos\left(4\pi^2 u^2\right)$

9.

$$\int_0^{1/2} \frac{1}{1+4x^2} dx$$

is

$a)\ \frac{\pi}{8}$ $b)\ \frac{\pi}{4}$ $c)\ \frac{\pi}{2}$ $d)\ \pi$ $e)$ undefined

10.

$$\lim_{x\to 1-} \frac{|x-1|}{x-1}$$

is

$a)\ 0$ $b)\ -1$ $c)\ 1$ $d)\ \infty$ $e)$ nonexistent

11.

$$\int_0^{\pi} \frac{d}{dx} e^{\cos(x)} dx$$

is

$a)\ \pi - \frac{1}{e}$ $b)\ \frac{1}{e} - e$ $c)\ e-1$ $d)\ 0$ $e)\ e^2+\pi$

12.

$$\int_{\ln(2)}^{\ln(3)} e^{2x} dx$$

is

$a)\ \frac{1}{8}$ $b)\ 5$ $c)\ \frac{5}{2}$ $d)\ -\frac{1}{4}$ $e)\ \frac{3}{8}$

13.

$$\int_0^1 \sin\left(3\pi t\right) dt$$

is

$a)\ \frac{3\pi}{2}$ $b)\ \frac{2}{3}\pi$ $c)\ 0$ $d)\ \frac{2}{3\pi}$ $e)\ -\frac{3\pi}{2}$

14. The area enclosed by the graphs of $y = x^2$ and $y = x$ is

$a)\ \frac{1}{6}$ $b)\ \frac{1}{3}$ $c)\ \frac{1}{2}$ $d)\ \frac{5}{6}$ $e)\ 1$

15.

$$\frac{d}{dx}\left(x^2 \sin(2x)\right)$$

is

$a)\ 4x\cos(2x)$ $b)\ 2x\sin(2x)+x^2\cos(2)$ $c)\ 2x\sin(2x)+x^2\cos(2x)$
$d)\ 2x\sin(2x)+2x^2\cos(2x)$ $e)\ 2x\sin(2x)-x^2\cos(2x)$

16.

$$\frac{d}{dx}\left(\frac{3x+4}{x^2+6}\right)$$

is

$a)\ \dfrac{3}{2x}$ $b)\ \dfrac{9x^2+8x+12}{(x^2+6)^2}$ $c)\ \dfrac{-3x^2-8x+18}{(x^2+6)^2}$
$d)\ \dfrac{9x^2+8x+12}{x^2+6}$ $e)\ \dfrac{3x^2+8x+18}{(x^2+6)^2}$

17.

$$\frac{d}{dx}\arcsin(4x)$$

is

$a)\ \dfrac{4}{\sqrt{1-16x^2}}$ $b)\ 4\arccos(4x)$ $c)\ \dfrac{1}{\sqrt{1-4x^2}}$
$d)\ \dfrac{4}{\sqrt{1+(4x)^2}}$ $e)\ -4\arcsin^2(4x)$

18.

$$\lim_{x\to 0+}\sqrt{x}\ln(x)$$

is

$a)\ 1$ $b)\ 0$ $c)\ +\infty$ $d)\ -\infty$ $e)$ nonexistent

19.

$$\lim_{x\to\pi}\frac{2\cos(x)+2-(x-\pi)^2}{(x-\pi)^2}$$

is

$a)\ 1$ $b)\ -1$ $c)\ 0$ $d)\ +\infty$ $e)\ -\infty$

20.

$$\lim_{x\to+\infty}\frac{e^{2x}}{x^2}$$

is

$a)\ +\infty$ $b)\ -\infty$ $c)\ 1$ $d)\ 0$ $e)$ nonexistent

Solutions

1.We have

$$f'(x) = \frac{d}{dx}\sin^2(2x) = 4\sin(2x)\cos(2x)$$

so that

$$f'(\pi/8) = 4\sin\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{4}\right) = 4\left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}}\right) = 2.$$

Therefore, the answer is **c).**

2.

$$\lim_{x\to+\infty}\frac{4x^3-5x}{x^3+6x-6} = \lim_{x\to+\infty}\frac{x^3\left(4-\frac{5}{x^2}\right)}{x^3\left(1+\frac{6}{x^2}-\frac{6}{x^3}\right)} = \lim_{x\to+\infty}\frac{4-\frac{5}{x^2}}{1+\frac{6}{x^2}-\frac{6}{x^3}} = 4.$$

Therefore, the answer is **c).**

3. We have

$$\frac{dy}{dx} = \frac{d}{dx}\left(2x^3-4x\right) = 6x^2-4,$$

so that

$$\frac{dy}{dx}(2) = 20.$$

Therefore, the tangent line is the graph of the equation

$$y = y(2) + \frac{dy}{dx}(2)(x-2) = 8 + 20(x-2) = 20x-32.$$

Therefore, the answer is **e).**

4.

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx}(\sin(3x)\cos(2x)) \\ &= 3\cos(3x)\cos(2x) - 2\sin(3x)\sin(2x).\end{aligned}$$

Therefore, the answer is **e).**

5.

$$f'(x) = \frac{d}{dx}\left((x-1)^2\sin(x)\right) = 2(x-1)\sin(x) + (x-1)^2\cos(x),$$

so that $f'(0) = -1$. Therefore, the answer is **b).**

6. We have

$$v(t) = \frac{d}{dt}\left(t^3 - t^2 + 4t + 6\right) = 3t^2 - 2t + 4,$$

and

$$a(t) = \frac{dv}{dt} = 6t - 2.$$

Therefore, $a(2) = 10$. Thus, the answer is **d).**

7. We have

$$f'(x) = 3x^2 - 3 = 3\left(x^2 - 1\right) = 0 \Leftrightarrow x = 1 \text{ or } x = -1.$$

We also have

$$f''(x) = 6x \Rightarrow f''(1) = 6 > 0 \text{ and } f''(-1) = -1.$$

By the second derivative test for local extrema, f has a local maximum at -1. Thus, the answer is **c).**

8. By the Fundamental Theorem of Calculus,

$$\frac{d}{dx}\int_2^x \cos\left(4\pi^2 u^2\right) du = \cos\left(4\pi^2 x^2\right).$$

Therefore the answer is **d)**

9. We have

$$\int_0^{1/2} \frac{1}{1+4x^2} dx = \frac{1}{2}\arctan(2x)\Big|_0^{1/2} = \frac{1}{2}\arctan(1) - \frac{1}{2}\arctan(0) = \frac{\pi}{8}.$$

Therefore the answer is **a)**

10.

$$\lim_{x\to 1-} \frac{|x-1|}{x-1} = \lim_{x\to 1-} \frac{-(x-1)}{(x-1)} = \lim_{x\to 1-} (-1) = -1.$$

Therefore the answer is **b)**

11. By the Fundamental Theorem of Calculus,

$$\int_0^{\pi} \frac{d}{dx} e^{\cos(x)} dx = e^{\cos(x)}\Big|_0^{\pi} = e^{\cos(\pi)} - e^{\cos(0)} = e^{-1} - e = \frac{1}{e} - e.$$

Therefore the answer is **b)**

12.

$$\int_{\ln(2)}^{\ln(3)} e^{2x} dx = \frac{1}{2}e^{2x}\Big|_{\ln(2)}^{\ln(3)} = \frac{1}{2}e^{2\ln(3)} - \frac{1}{2}e^{2\ln(2)} = \frac{1}{2}\left(3^2 - 2^2\right) = \frac{5}{2}$$

Therefore the answer is **c)**

13.

$$\begin{aligned}\int_0^1 \sin(3\pi t)\,dt = -\frac{1}{3\pi}\cos(3\pi t)\Big|_0^1 &= -\frac{1}{3\pi}(\cos(3\pi)-\cos(0)) \\ &= -\frac{1}{3\pi}(-1-1) = \frac{2}{3\pi}\end{aligned}$$

Therefore the answer is **d)**

14. The area enclosed by the graphs of $y = x^2$ and $y = x$ is

$$\int_0^1 \left(x - x^2\right) dx = \frac{1}{2}x^2 - \frac{1}{3}x^3\Big|_0^1 = \frac{1}{2} - \frac{1}{3} = \frac{1}{6}$$

Therefore the answer is **a)**

15.

$$\frac{d}{dx}\left(x^2\sin(2x)\right) = 2x\sin\left(2x^2\right) + x^2\left(2\cos(2x)\right) = 2x\sin(2x) + 2x^2\cos(2x).$$

Therefore the answer is **d)**

16.

$$\frac{d}{dx}\left(\frac{3x+4}{x^2+6}\right) = \frac{3\left(x^2+6\right) - (3x+4)(2x)}{\left(x^2+6\right)^2} = \frac{-3x^2-8x+18}{\left(x^2+6\right)^2}$$

Therefore the answer is **c)**

17.

$$\frac{d}{dx}\arcsin(4x) = \frac{d}{du}\arcsin(u)\Big|_{u=4x}(4) = \frac{4}{\sqrt{1-(4x)^2}} = \frac{4}{\sqrt{1-16x^2}}.$$

Therefore the answer is **a)**

18.

$$\lim_{x\to 0+}\sqrt{x}\ln(x) = \lim_{x\to 0+}\frac{\ln(x)}{x^{-1/2}} = \lim_{x\to 0+}\frac{\frac{1}{x}}{-\frac{1}{2}x^{-3/2}} = -2\lim_{x\to 0+}\left(x^{1/2}\right) = 0.$$

Therefore the answer is **b)**

19.

$$\begin{aligned}\lim_{x\to\pi}\frac{2\cos(x)+2-(x-\pi)^2}{(x-\pi)^2} &= \lim_{x\to\pi}\frac{-2\sin(x)-2(x-\pi)}{2(x-\pi)} \\ &= \lim_{x\to\pi}\frac{-2\cos(x)-2}{2} = -\cos(\pi)-1 = 0.\end{aligned}$$

Therefore the answer is **c)**

20.

$$\lim_{x\to+\infty}\frac{e^{2x}}{x^2} = \lim_{x\to+\infty}\frac{2e^{2x}}{2x} = \lim_{x\to+\infty}\frac{4e^{2x}}{2} = +\infty.$$

Therefore the answer is **a)**