Title: The Combinatorics of Pattern Avoidance in Permutations and Words

Abstract: Given a permutation $\tau = \tau_1 \dots \tau_k$, we say that τ occurs in permutation $\sigma = \sigma_1 \dots \sigma_n$ there is a subsequence $1 \leq i_1 < \dots < i_k \leq n$ such that $\sigma_{i_1}\sigma_{i_2} \dots \sigma_{i_k}$ has the same relative order as $\tau_1\tau_2 \dots \tau_k$. For example, 1 3 2 occurs in 2 1 5 4 3 but 1 2 3 does not occur in 2 1 5 4 3. We say that a permutation $\sigma = \sigma_1 \dots \sigma_n$ avoids $\tau = \tau_1 \dots \tau_k$ if τ does not occur in σ. For any given permutation $\tau = \tau_1 \dots \tau_k$, let $S_n(\tau)$ denote the number of permutations of length n that avoid τ. We then say that permutations $\tau = \tau_1 \dots \tau_k$ and $\gamma = \gamma_1 \dots \gamma_k$ are Wilf equivalent if $S_n(\tau) = S_n(\gamma)$ for all n.

There has been a large literature on pattern avoidance in permutations and words. We will give a brief survey of some results on Wilf equivalence for permutations and words. In particular, we will talk about a recent variation of the notion of Wilf equivalence for words relative to the factor order. Given two words $v = v_1 \cdots v_m$ and $w = w_1 \cdots w_m$ over the positive integers P, we say that v embeds into w at starting position i if $v_j \leq w_{i-1+j}$ for $j = 1, \dots, m$. Then we say that v is less than or equal to w in the factor order, $u \leq_F w$, if there is an i such that v embeds into w at starting at position i. We say that w avoids v if it is not the case that $v \leq_F w$. We show that for any word $v \in P^*$, the language E_v of all words w such that $v \leq_F w$ and the language A_v of all words w that avoid v are accepted by finite automaton. It follows that the weight generating functions

$$E_v(x,t) = \sum_{w=w_1\cdots w_n \in E_v} x^{w_1+\cdots+w_n} t^n$$

and

$$A_v(x,t) = \sum_{w=w_1\cdots w_n \in A_v} x^{w_1+\cdots+w_n} t^n$$

are rational functions.

We say that two words in $u, v \in P^*$ are Wilf equivalent if and only if $A_v(x,t) = A_u(x,t)$. Our results give a decision procedure to decide if two words u and v are Wilf equivalent. We have computed tables of the functions $A_v(x,t)$ and these tables yeild a number of interesting Wilf equivalences and lead to a number of natural bijective questions.