Math 150 Fall 2004
Group Final
Solutions

1.
$$\frac{d}{dx}\left(x^2 \arctan(x)\right) = 2x \arctan(x) + x^2\left(\frac{1}{x^2+1}\right)$$

2.
$$\frac{d}{dx}\left(\frac{x^2-1}{x^2+4}\right) = \frac{2x\left(x^2+4\right) - \left(x^2-1\right)2x}{\left(x^2+4\right)^2}$$
$$= \frac{2x^3+8x-2x^3+2x}{\left(x^2+4\right)^2} = \frac{10x}{\left(x^2+4\right)^2}$$

3.
$$\frac{d}{dx}\int_{\pi}^{x} e^{\cos^2(t)}\,dt = e^{\cos^2(x)}$$

4.
$$\frac{d}{dx}\sqrt{1+\sin^2(x)} = \frac{1}{2\sqrt{1+\sin^2(x)}}\left(2\sin(x)\cos(x)\right) = \frac{\sin(x)\cos(x)}{\sqrt{1+\sin^2(x)}}$$

5.
$$\left.\frac{d}{dx}\sinh(x)\right|_{x=\ln(2)} = \cosh(\ln(2)) = \frac{e^{\ln(2)} + e^{-\ln(2)}}{2}$$
$$= \frac{2+\frac{1}{2}}{2} = \frac{5}{4}$$

6.
$$\lim_{x\to 4}\frac{x^2-9}{x-2} = \frac{16-9}{4-2} = \frac{7}{2}$$

7. If $x \cong \pi/2$ and $x > \pi/2$,
$$\frac{\sin^2(x)}{\cos(x)} \cong \frac{1}{\cos(x)},$$
and $\cos(x) < 0$ so that
$$\lim_{x\to\pi/2+}\frac{\sin^2(x)}{\cos(x)} = \lim_{x\to\pi/2+}\frac{1}{\cos(x)} = -\infty.$$

8.
$$\lim_{x\to+\infty}\left(1+x^2\right)^{1/x} = \lim_{x\to+\infty}\exp\left(\frac{\ln\left(1+x^2\right)}{x}\right) = \exp\left(\lim_{x\to+\infty}\frac{\ln\left(1+x^2\right)}{x}\right)$$
$$= \exp\left(\lim_{x\to+\infty}\frac{\frac{1}{1+x^2}(2x)}{1}\right) = \exp(0) = 1.$$

9. Set $u = x^2 + 1$ so that $du = 2x dx$. Therefore,

$$\int x\sqrt{1+x^2}dx = \frac{1}{2}\int \sqrt{u}du = \frac{1}{2}\int u^{1/2}du$$
$$= \frac{1}{2}\left(\frac{u^{3/2}}{3/2}\right) + C = \frac{1}{3}\left(x^2+1\right)^{3/2} + C$$

10. Set $u = \sin(x)$ so that $du = \cos(x)\,dx$. Therefore,

$$\int \frac{\cos(x)}{1+\sin^2(x)}dx = \int \frac{1}{1+u^2}du$$
$$= \arctan(u) + C$$
$$= \arctan(\sin(x)) + C.$$

11.

$$\int_{1/2}^{1} \frac{d}{dx}\left(\arcsin(x)\right)dx = \arcsin(x)\big|_{1/2}^{1}$$
$$= \arcsin(1) - \arcsin(1/2)$$
$$= \frac{\pi}{2} - \frac{\pi}{6} = \frac{\pi}{3}$$

12. Set $u = \ln(x)$ so that $du = (1/x)\,dx$. Therefore,

$$\int \frac{\ln^2(x)}{x}dx = \int u^2 du = \frac{u^3}{3} + C = \frac{1}{3}\ln^3(x) + C$$

Thus,

$$\int_1^e \frac{\ln^2(x)}{x}dx = \frac{1}{3}\left(\ln(e)\right)^3 - \frac{1}{3}\left(\ln(1)\right)^3 = \frac{1}{3}$$