

Math 151 Fall 2004
Group Final

1. We set $u = x$ and $dv = e^{-x/2}$ so that

$$du = dx \text{ and } v = -2e^{-x/2}.$$

Therefore,

$$\begin{aligned} \int x e^{-x/2} dx &= \int u dv \\ &= uv - \int v du \\ &= x \left(-2e^{-x/2} \right) + 2 \int e^{-x/2} dx \\ &= -2x e^{-x/2} - 4e^{-x/2} + C \end{aligned}$$

2. We set $u = \arcsin(x)$ and $dv = dx$ so that

$$du = \frac{1}{\sqrt{1-x^2}} dx \text{ and } v = x.$$

Therefore,

$$\begin{aligned} \int \arcsin(x) dx &= \int u dv \\ &= uv - \int v du \\ &= \arcsin(x)(x) - \int x \left(\frac{1}{\sqrt{1-x^2}} \right) dx \\ &= x \arcsin(x) - \int \frac{x}{\sqrt{1-x^2}} dx. \end{aligned}$$

To evaluate the integral on the RHS we set $w = 1 - x^2$ so that $dw = -2x dx$. Thus,

$$\int \frac{x}{\sqrt{1-x^2}} dx = -\frac{1}{2} \int u^{-1/2} du = -\frac{1}{2} \left(\frac{u^{1/2}}{1/2} \right) = -\sqrt{1-x^2}.$$

Therefore,

$$\int \arcsin(x) dx = x \arcsin(x) + \sqrt{1-x^2} + C.$$

3.

$$\frac{x+23}{x^2-3x-10} = \frac{4}{x-5} - \frac{3}{x+2}$$

Therefore,

$$\int \frac{x+23}{x^2-3x-10} dx = 4 \ln(|x-5|) - 3 \ln(|x+2|) + C$$

4.

$$\begin{aligned} \lim_{n \rightarrow \infty} \frac{\frac{3^{n+1}}{(n+1)!}}{\frac{3^n}{n!}} &= \lim_{n \rightarrow \infty} \left(\frac{n!}{(n+1)!} \right) \left(\frac{3^{n+1}}{3^n} \right) \\ &= 3 \lim_{n \rightarrow \infty} \frac{1}{n+1} = 0 < 1. \end{aligned}$$

Therefore the series converges.

5.

$$\lim_{n \rightarrow \infty} \left| \frac{(x-2)^n}{2^n n^2} \right|^{1/n} = |x-2| \lim_{n \rightarrow \infty} \frac{1}{2n^{2/n}} = \frac{1}{2} |x-2|.$$

Therefore, the series converges absolutely if $|x-2| < 2$ and diverges if $|x-2| > 2$. Thus, the radius of convergence is 2 and the open interval of convergence is $(0, 4)$.

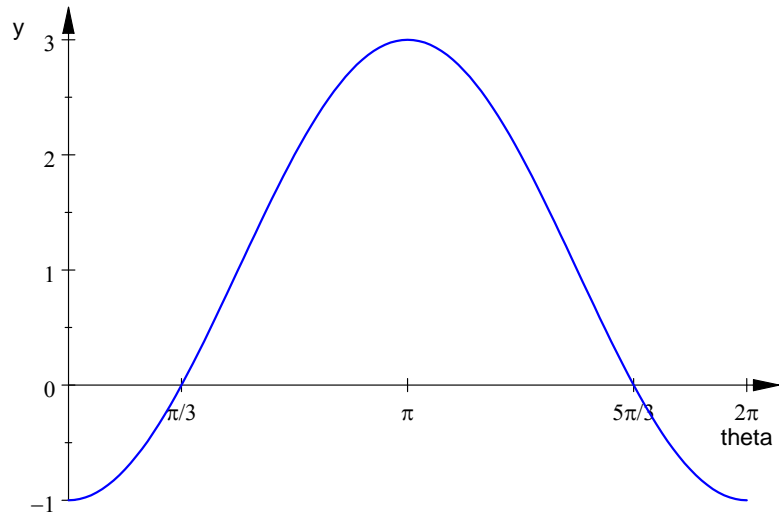
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$$\begin{aligned} f(x) &= (x+1)^{1/3} \Rightarrow f(0) = 1, \\ f'(x) &= \frac{1}{3}(x+1)^{-2/3} \Rightarrow f'(0) = \frac{1}{3}, \\ f''(x) &= -\frac{2}{9}(x+1)^{-5/3} \Rightarrow f''(0) = -\frac{2}{9}, \\ f'''(x) &= \frac{10}{27}(x+1)^{-8/3} \Rightarrow f'''(0) = \frac{10}{27}. \end{aligned}$$

Therefore,

$$P_3(x) = 1 + \frac{1}{3}x - \frac{1}{9}x^2 + \frac{10}{6(27)}x^3 = 1 + \frac{1}{3}x - \frac{1}{9}x^2 + \frac{5}{81}x^3$$

7.a)



b)

