Last Name:
Name:
Instructor:

Math 150
Group Final (Fall 2005)
Version A

You are not allowed to use notes, books, calculators, personal stereos or cell phones. You have exactly 2 hours.
This is a multiple-choice exam. You will use only the provided blank work packet for the calculations. At the end of the exam you will turn in the scantron, the question packet and the work packet with your name on each item. Any missing item may result in disciplinary action and failure in the course.

1.

$$\lim_{x\to 2+} \frac{x-4}{(x^2+x-6)}$$

is

a) 0 *b)* $+\infty$ *c)* $-\infty$ *d)* $\frac{1}{4}$ *e)* nonexistent

2.

$$\lim_{x\to 2-} \frac{\left|x^2-4\right|}{x-2}$$

is

a) 4 *b)* -4 *c)* 2 *d)* -2 *e)* nonexistent

3.

$$\lim_{x\to 0} \frac{\sin(3x)-3x}{x^3}$$

is

a) $+\infty$ *b)* $-\infty$ *c)* -9 *d)* $-\frac{9}{2}$ *e)* $-\frac{3}{2}$

4.

$$\lim_{x\to +\infty} x^2 e^{-x/2}$$

is

a) $+\infty$ *b)* $-\infty$ *c)* 0 *d)* 1 *e)* nonexistent

5.

$$\lim_{x\to +\infty} \left(1+\frac{1}{2x}\right)^x$$

is

a) $\frac{1}{2}$ *b)* $e^{1/2}$ *c)* e^{-2} *d)* e^2 *e)* $e^{-1/2}$

6. If $f(x) = \dfrac{x^2-4}{x^2+4}$ then $f'(x)$ is

a) $\dfrac{4x}{(x^2+4)^2}$ *b)* $\dfrac{16x}{x^2+4}$ *c)* $\dfrac{4x}{(x^2+4)^2}$ *d)* $\dfrac{16x}{(x^2+4)^2}$ *e)* $\dfrac{-16}{(x^2+4)^2}$

7. If $f(t) = 4\sin\left(\dfrac{t}{2}\right)$ then $f'\left(-\dfrac{\pi}{2}\right)$ is

a) $\sqrt{2}$ *b)* $-\sqrt{2}$ *c)* $\dfrac{1}{\sqrt{2}}$ *d)* $-\dfrac{1}{\sqrt{2}}$ *e)* $\dfrac{\sqrt{3}}{2}$

8. .If $f(x) = (x^2-1)^{1/3}$ then $f'(x)$ is

a) $\dfrac{1}{3}(x^2-1)^{-2/3}$ *b)* $\dfrac{1}{3}(x^2-1)^{2/3}$ *c)* $\dfrac{x}{3(x^2-1)^{2/3}}$

d) $\dfrac{2x}{3(x^2-1)^{2/3}}$ *e)* $\dfrac{2x}{3(x^2-1)^{1/3}}$

9.

$$\frac{d}{dx}\arccos(3x)$$

is

a) $\dfrac{1}{\sqrt{1-9x^2}}$ *b)* $3\arcsin(3x)$ *c)* $\dfrac{-3}{\sqrt{1-9x^2}}$

d) $\dfrac{3}{\sqrt{1+9x^2}}$ *e)* $-3\arcsin^{-2}(3x)$

10. If $f(x) = x^2\ln(x)$, then $f'(e)$ is

a) 1 *b)* $3e$ *c)* $2e$ *d)* e *e)* $4e$

11. The equation of the tangent line to the graph of $f(x) = \sqrt{7+x^2}$ at $(3, f(3))$ is

a) $y = \dfrac{4}{3}x - \dfrac{7}{3}$ *b)* $y = -\dfrac{3}{4}x + \dfrac{7}{4}$ *c)* $y = \dfrac{7}{4}x + 1$

d) $y = \dfrac{3}{4}x + \dfrac{7}{4}$ *e)* $y = \dfrac{3}{7}x - \dfrac{4}{7}$

12. Let $f(x) = x^{1/4}$. The approximation to $(15.8)^{1/4}$ via the linear approximation to f based at 16 (the linearization of f at 16) is

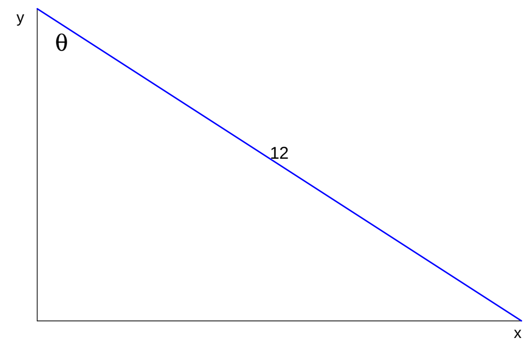
a) $16 + \dfrac{1}{48}(0.2)$ *b)* $16 - \dfrac{0.2}{32}$ *c)* $16 + \dfrac{0.2}{32}$ *d)* $2 - \dfrac{0.2}{32}$ *e)* $2 + \dfrac{0.2}{32}$

13. If the position of a particle moving along the x-axis is $f(t) = e^{-t}\sin(t)$ at time t, then the acceleration at t is

a) $-e^{-t}\sin(t) + e^{-t}\cos(t)$ *b)* $e^{-t}\sin(t) - e^{-t}\cos(t)$

c) $-2e^{-t}\cos(t)$ *d)* $2e^{-t}\cos(t)$ *e)* $-2e^{-t}\sin(t)$

14. Assume that a ladder which is 12 feet long is leaning against a wall and its base is sliding away from the wall at the rate of 4 ft/sec.



Determine the rate at which the angle θ (in radians) between the top of the ladder and the wall is increasing at the instant $\theta = \pi/3$:

a) 8 *b*) 2 *c*) $\dfrac{4}{\sqrt{3}}$ *d*) $\dfrac{2}{3}$ *e*) 1

15. Let $f(x) = x^2 (x-4)^{2/3}$. The absolute maximum of f on the interval $[-1, 4]$ is

a) 10 *b*) 9 *c*) 12 *d*) $5^{2/3}$ *e*) $4^{2/3}$

16. Let $f(x) = \dfrac{1}{12}x^4 + \dfrac{1}{6}x^3 - 3x^2$. The graph of f is concave up on

a) $(-3, 2)$ *b*) $(-2, 3)$ *c*) $(-\infty, -2)$
d) $(-\infty, -3)$ and $(2, +\infty)$ *e*) $(-\infty, -2)$ and $(1, +\infty)$

17. Let

$$f(x) = -x^2 + \frac{1}{x}$$

The function f attains its absolute maximum on the interval $(-\infty, 0)$ at

a) -2 *b*) $-2^{-1/3}$ *c*) -2^3 *d*) $-2^{-1/2}$ *e*) -3

18.

$$\int_1^4 \frac{d}{dx}\left(\frac{x-1}{x^2+1}\right) dx$$

is

a) $\dfrac{2}{17}$ *b*) 3 *c*) 0 *d*) -3 *e*) $\dfrac{3}{17}$

19.

$$\frac{d}{dx}\int_{-1}^{x}\sqrt{u^4+16}du$$

is

a) $\sqrt{x^4+16}-\sqrt{17}$ *b)* $\frac{2x^3}{\sqrt{x^4+16}}$ *c)* $\sqrt{x^4+16}$

d) $\sqrt{17}-\sqrt{x^4+16}$ *e)* $\sqrt{u^4+16}$

20.

$$\int_{\pi^2/36}^{\pi^2/4}\frac{\cos\left(\sqrt{x}\right)}{\sqrt{x}}dx$$

is

a) 0 *b)* $\frac{\pi}{4}$ *c)* 1 *d)* 2 *e)* π

21.

$$\int_{1}^{e^2}\frac{1}{4x}dx$$

is

a) $\frac{1}{4}$ *b)* $\frac{e}{2}$ *c)* e^2-1 *d)* $\frac{e}{4}$ *e)* $\frac{1}{2}$

22.

$$\int_{-1/3}^{1/3}\frac{1}{1+9x^2}dx$$

is

a) $\frac{\pi}{3}$ *b)* $\frac{\pi}{6}$ *c)* $\frac{\pi}{4}$ *d)* $\frac{\pi}{2}$ *e)* π

23. The area of the region enclosed by the graphs of $y=x^2$ and $y=2x-x^2$ is

a) $\frac{1}{6}$ *b)* $\frac{1}{3}$ *c)* $\frac{1}{2}$ *d)* $\frac{5}{6}$ *e)* 1

24. Assume that the velocity of an object moving along the x-axis is $\sin\left(\frac{\pi t}{4}\right)$ at time t. The distance traveled by the object over the time interval $[0,6]$ is

a) $\frac{8}{\pi}$ *b)* $\frac{\pi}{12}$ *c)* $\frac{12}{\pi}$ *d)* 4 *e)* 12

25. Assume that

$$\frac{dy}{dt} = e^{-t/2} \text{ and } y(1) = -2.$$

Then $y(t)$ isThen $y(t)$ is

$a)\ -2 - e^{-t/2} + e^{-1/2}$ $\quad b)\ 2 + e^{-t/2} - e^{-1/2}$ $\quad c)\ -2 + \frac{1}{2}e^{-t/2} - \frac{1}{2}e^{-1/2}$

$d)\ -2 + 2e^{-t/2} - 2e^{-1/2}$ $\quad e)\ -2e^{-t/2} + 2e^{-1/2} - 2$

Solutions

1.

$$\lim_{x\to 2+} \frac{x-4}{(x^2+x-6)} = \lim_{x\to 2+}\left(\left(\frac{x-4}{x+3}\right)\left(\frac{1}{x-2}\right)\right) = \left(-\frac{2}{5}\right)\lim_{x\to 2+}\frac{1}{x-2} = -\infty.$$

The answer is **c.**

2.

$$\lim_{x\to 2-}\frac{\left|x^2-4\right|}{x-2} = \lim_{x\to 2-}\frac{|(x-2)(x+2)|}{x-2} = \lim_{x2-}\frac{-(x-2)(x+2)}{x-2} = -\lim_{x\to 2}(x+2) = -4$$

The answer is **b.**

3. By L'Hôpital's rule,

$$\lim_{x\to 0}\frac{\sin(3x)-3x}{x^3} = \lim_{x\to 0}\frac{3\cos(3x)-3}{3x^2} = \lim_{x\to 0}\frac{-9\sin(3x)}{6x} = -\frac{9}{2}\lim_{x\to 0}\frac{\sin(3x)}{3x} = -\frac{9}{2}$$

The answer is **d.**

4. By L'Hôpital's rule,

$$\begin{aligned}\lim_{x\to+\infty} x^2e^{-x/2} = \lim_{x\to+\infty}\frac{x^2}{e^{-x/2}} = \lim_{x\to+\infty}\frac{2x}{-\frac{1}{2}e^{-x/2}} &= \lim_{x\to+\infty}\frac{-4x}{e^{-x/2}} \\ &= \lim_{x\to+\infty}\frac{-4}{-\frac{1}{2}e^{-x/2}} = \lim_{x\to+\infty}\frac{8}{e^{-x/2}} = 0.\end{aligned}$$

The answer is **c.**

5. We have

$$\lim_{x\to+\infty}\left(1+\frac{1}{2x}\right)^x = \lim_{x\to+\infty}\exp\left(x\ln\left(1+\frac{1}{2x}\right)\right) = \exp\left(\lim_{x\to+\infty} x\ln\left(1+\frac{1}{2x}\right)\right).$$

By L'Hôpital's rule,

$$\begin{aligned}\lim_{x\to+\infty} x\ln\left(1+\frac{1}{2x}\right) = \lim_{x\to+\infty}\frac{\ln\left(1+\frac{1}{2x}\right)}{\frac{1}{x}} &= \lim_{x\to+\infty}\frac{\frac{2x}{2x+1}\left(-\frac{1}{2x^2}\right)}{-\frac{1}{x^2}} \\ &= \lim_{x\to+\infty}\frac{x}{2x+1} = \frac{1}{2}.\end{aligned}$$

Therefore,

$$\lim_{x\to+\infty}\left(1+\frac{1}{2x}\right)^{x}=\exp\left(\lim_{x\to+\infty}x\ln\left(1+\frac{1}{2x}\right)\right)=\exp\left(\frac{1}{2}\right)=e^{1/2}.$$

The answer is **b.**

6.

$$f'(x)=\frac{d}{dx}\left(\frac{x^2-4}{x^2+4}\right)=\frac{2x\left(x^2+4\right)-\left(x^2-4\right)2x}{\left(x^2+4\right)^2}=\frac{16x}{\left(x^2+4\right)^2}$$

The answer is **d.**

7.

$$f'(t)=\frac{d}{dt}\left(4\sin\left(\frac{t}{2}\right)\right)=4\left(\cos\left(\frac{t}{2}\right)\left(\frac{1}{2}\right)\right)=2\cos\left(\frac{t}{2}\right)$$

Therefore,

$$f'\left(-\frac{\pi}{2}\right)=2\cos\left(-\frac{\pi}{4}\right)=2\left(\frac{\sqrt{2}}{2}\right)=\sqrt{2}$$

The answer is **a.**

8.

$$f'(x)=\frac{d}{dx}\left(x^2-1\right)^{1/3}=\frac{1}{3}\left(x^2-1\right)^{-2/3}(2x)=\frac{2x}{3\left(x^2-1\right)^{2/3}}.$$

The answer is **d.**

9.

$$\frac{d}{dx}\arccos(3x)=-\frac{1}{\sqrt{1-(3x)^2}}(3)=-\frac{3}{\sqrt{1-9x^2}}$$

The answer is **c.**

10. We have

$$f'(x)=\frac{d}{dx}\left(x^2\ln(x)\right)=2x\ln(x)+x^2\left(\frac{1}{x}\right)=2x\ln(x)+x.$$

Therefore,

$$f'(e)=2e\ln(e)+e=2e+e=3e.$$

The answer is **b.**

11. We have

$$f'(x)=\frac{d}{dx}\left(7+x^2\right)^{1/2}=\frac{1}{2}\left(7+x^2\right)^{-1/2}(2x)=\frac{x}{\sqrt{7+x^2}}.$$

Therefore,

$$f(3)=4\text{ and }f'(3)=\frac{3}{4}.$$

Thus,

$$y=4+\frac{3}{4}(x-3)=\frac{3}{4}x+\frac{7}{4}$$

The answer is **d.**

12. We have

$$f'(x) = \frac{d}{dx}x^{1/4} = \frac{1}{4}x^{-3/4} = \frac{1}{4x^{3/4}}.$$

Therefore,

$$f(16) = 16^{1/4} = 2$$

and

$$f'(16) = \left.\frac{1}{4}x^{-3/4}\right|_{x=16} = \left.\frac{1}{4x^{3/4}}\right|_{x=16} = \frac{1}{4(8)} = \frac{1}{32},$$

so that

$$L(x) = f(16) + f'(16)(x-16) = 2 + \frac{1}{32}(x-16).$$

Therefore,

$$f(15.8) \cong L(15.8) = 2 - \frac{0.2}{32}$$

The answer is **d.**

13. We have

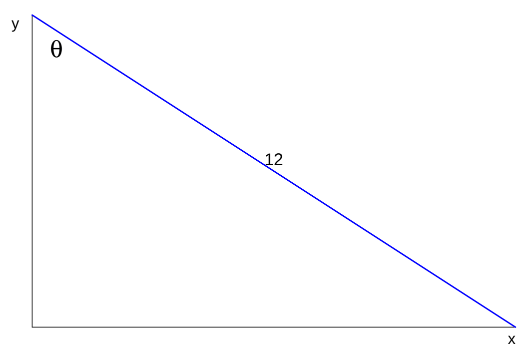
$$v(t) = \frac{d}{dt}e^{-t}\sin(t) = -e^{-t}\sin(t) + e^{-t}\cos(t),$$

and

$$a(t) = \frac{dv}{dt} = e^{-t}\sin(t) - e^{-t}\cos(t) - e^{-t}\cos(t) - e^{-t}\sin(t) = -2e^{-t}\cos(t)$$

The answer is **c.**

14.



We have

$$\sin(\theta) = \frac{x}{12} \Rightarrow \cos(\theta)\frac{d\theta}{dt} = \frac{1}{12}\left(\frac{dx}{dt}\right) = \frac{1}{12}(4) = \frac{1}{3}.$$

Therefore,

$$\frac{d\theta}{dt} = \frac{1}{3\cos(A)}.$$

At the instant $\theta = \pi/3$,

$$\frac{d\theta}{dt} = \frac{1}{3\cos(\pi/3)} = \frac{2}{3} \text{ radians/sec.}$$

The answer is **d.**

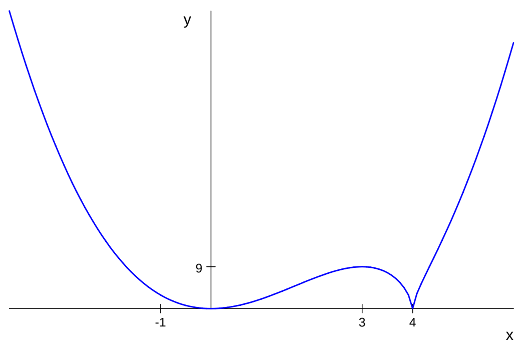
15. We have

$$f'(x) = \frac{d}{dx}x^2(x-4)^{2/3} = 2x(x-4)^{2/3} + x^2\left(\frac{2}{3}(x-4)^{-1/3}\right)$$
$$= 2x(x-4)^{2/3} + \frac{2x^2}{3(x-4)^{1/3}}$$
$$= \frac{6x(x-4)+2x^2}{(x-4)^{1/3}} = \frac{8x^2-24x}{(x-4)^{1/3}} = \frac{8x(x-3)}{(x-4)^{1/3}}.$$

Therefore, the critical points of f are 0, 3 and 4. We have

$$f(-1) = 5^{2/3},\ f(0) = f(4) = 0 \text{ and } f(3) = 9.$$

Therefore, the absolute maximum of f on the interval $[-1, 4]$ is 9. The answer is **b.**



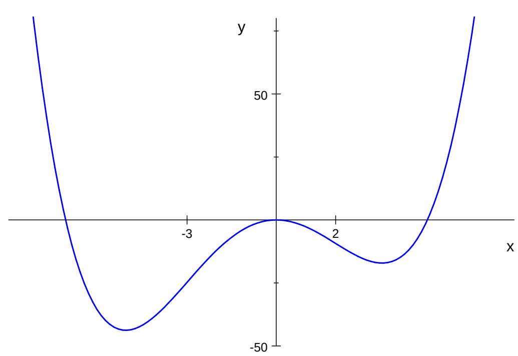
16. We have

$$f'(x) = \frac{d}{dx}\left(\frac{1}{12}x^4 + \frac{1}{6}x^3 - 3x^2\right) = \frac{1}{3}x^3 + \frac{1}{2}x^2 - 6x,$$

and

$$f''(x) = \frac{d}{dx}\left(\frac{1}{3}x^3 + \frac{1}{2}x^2 - 6x\right) = x^2 + x - 6 = (x+3)(x-2)$$

Therefore, $f''(x) > 0$ if $x \in (-\infty, -3)$ or $x \in (2, +\infty)$. Thus, the graph of f is concave up on the intervals $(-\infty, -3)$ and $(2, +\infty)$. The answer is **d.**



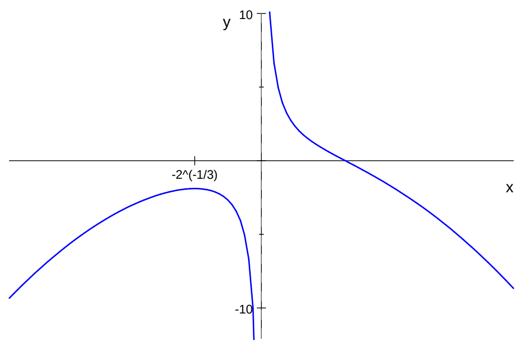
17. We have

$$f'(x) = \frac{d}{dx}\left(-x^2 + \frac{1}{x}\right) = -2x - \frac{1}{x^2} = \frac{-2x^3 - 1}{x^2} = -\frac{2x^3 + 1}{x^2}.$$

Therefore $f'(x) = 0$ if

$$2x^3 + 1 = 0 \Leftrightarrow x^3 = -\frac{1}{2} \Leftrightarrow x = -\frac{1}{2^{1/3}}$$

We have $f'(x) > 0$ if $x < -2^{-1/3}$ and $f'(x) < 0$ if $-2^{-1/3} < x < 0$. Therefore, f is increasing on $(-\infty, -2^{-1/3}]$ and decreasing on $[-2^{-1/3}, 0)$. Thus, f attains its absolute maximum on $(-\infty, 0)$ at $-2^{-1/3}$. The answer is **b.**



18.

$$\int_1^4 \frac{d}{dx}\left(\frac{x-1}{x^2+1}\right) dx = \left.\frac{x-1}{x^2+1}\right|_{x=1}^4 = \frac{4-1}{16+1} - \frac{1-1}{1+1} = \frac{3}{17}$$

The answer is **e.**

19.

$$\frac{d}{dx}\int_{-1}^{x} \sqrt{u^4+16}du = \sqrt{x^4+16}$$

The answer is **c.**

20. We set $u = \sqrt{x}$ so that $du = 1/(2\sqrt{x})\, dx$. Thus,

$$\int_{\pi^2/36}^{\pi^2/4} \frac{\cos(\sqrt{x})}{\sqrt{x}} dx = 2\int_{\pi/6}^{\pi/2} \cos(u)\, du = 2\left(\sin(u)|_{\pi/6}^{\pi/2}\right) = 2\left(1 - \frac{1}{2}\right) = 1$$

The answer is **c.**

21.

$$\int_1^{e^2} \frac{1}{4x} dx = \frac{1}{4}\left(\ln(|x|)|_{x=1}^{e^2}\right) = \frac{1}{4}\ln\left(e^2\right) = \frac{2}{4} = \frac{1}{2}$$

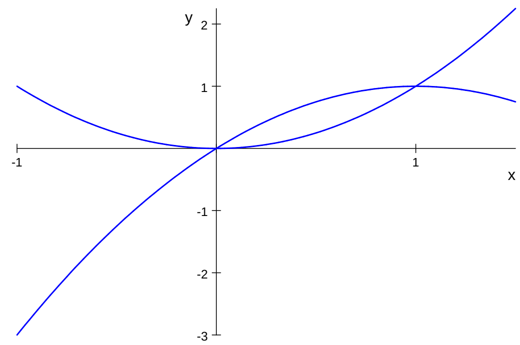
The answer is **e**.

22. We set $u = 3x$ so that $du = 3dx$. Thus,

$$\int_{-1/3}^{1/3} \frac{1}{1+9x^2} dx = \frac{1}{3}\int_{-1}^{1} \frac{1}{1+u^2} du = \frac{1}{3}\left(\arctan(u)|_{-1}^{1}\right) = \frac{1}{3}\left(\frac{\pi}{4} + \frac{\pi}{4}\right) = \frac{\pi}{6}.$$

The answer is **b.**

23.



We have

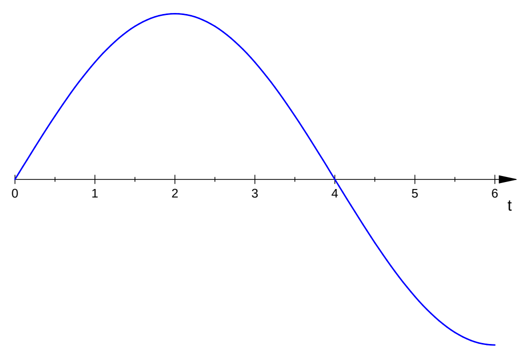
$$x^2 = 2x - x^2 \Leftrightarrow 2x^2 - 2x = 0 \Leftrightarrow x\,(x-1) = 0 \Leftrightarrow x = 0 \text{ or } x = 1.$$

The area of the region enclosed by the graphs of $y = x^2$ and $y = 2x - x^2$ is

$$\int_0^1 \left(\left(2x - x^2\right) - x^2\right) dx = \int_0^1 \left(2x - 2x^2\right) dx = 2\left(\frac{x^2}{2} - \frac{x^3}{3}\bigg|_0^1\right) = 2\left(\frac{1}{2} - \frac{1}{3}\right) = \frac{1}{3}$$

The answer is **b.**

24.



The distance traveled by the object over the time interval $[0, 6]$ is

$$\begin{aligned}
\int_0^4 \sin\left(\frac{\pi t}{4}\right) dt - \int_4^6 \sin\left(\frac{\pi t}{4}\right) dt &= \frac{4}{\pi}\left(-\cos\left(\frac{\pi t}{4}\right)\Big|_0^4\right) - \frac{4}{\pi}\left(-\cos\left(\frac{\pi t}{4}\right)\Big|_4^6\right) \\
&= \frac{4}{\pi}\left(-\cos(\pi) + \cos(0)\right) - \frac{4}{\pi}\left(-\cos\left(\frac{3\pi}{2}\right) + \cos(\pi)\right) \\
&= \frac{8}{\pi} + \frac{4}{\pi} = \frac{12}{\pi}
\end{aligned}$$

The answer is **c.**

25.

$$\begin{aligned}
y(t) = y(1) + \int_1^t \frac{dy}{du} du = -2 + \int_1^t e^{-u/2} du &= -2 - 2\int_{-1/2}^{-t/2} e^w dw \\
&= -2 - 2e^{-t/2} + 2e^{-1/2} \\
&= -2e^{-t/2} + 2e^{-1/2} - 2
\end{aligned}$$

The answer is **e.**