Study Guide and Sample Problems for the Fall 2005 Math 151 Group Final

Answers

Revised: August 8, 2005

1.

a. $\int x\cos(x)dx = \cos(x) + x\sin(x) + C$

b. $\int x^2\sin(x)dx = \left(2 - x^2\right)\cos(x) + 2x\sin(x) + C$

c. $\int x^2 e^{-x}dx = \left(-x^2 - 2x - 2\right)e^{-x} + C$

d. $\int x^{1/3}\ln(x)dx = \frac{3}{4}x^{4/3}\ln(x) - \frac{9}{16}x^{4/3} + C$

e. $\int \sin^2(x)\,dx = -\frac{1}{2}\cos(x)\sin(x) + \frac{1}{2}x + C$

f. $\int \cos^2(x)\,dx = \frac{1}{2}\cos(x)\sin(x) + \frac{1}{2}x + C$

g. $\int \cos^3(x)\sin^2(x)\,dx = \frac{1}{3}\sin^3(x) - \frac{1}{5}\sin^5(x) + C$

h. $\displaystyle\int \frac{1}{(x-1)(x+3)}dx = \frac{1}{4}\ln|x-1| - \frac{1}{4}\ln|x+3| + C$

i. $\displaystyle\int \frac{x}{x^2+9}dx = \frac{1}{2}\ln\left(x^2+9\right) + C$

j. $\displaystyle\int \frac{1}{x^2-2x+5}dx = \frac{1}{2}\arctan\left(\frac{x-1}{2}\right) + C$

k. $\displaystyle\int \frac{x}{(x^2+1)^2}dx = -\frac{1}{2(x^2+1)} + C$

l. $\displaystyle\int \frac{2x^2+7x}{x^2+6x+9} = 2x + \frac{3}{x+3} - 5\ln|x+3| + C$

2.

a. $\displaystyle\int_1^\infty e^{-x}dx = e^{-1}$

b. $\displaystyle\int_0^\infty xe^{-x}dx = 1$

c. $\displaystyle\int_0^\infty \cos(x)\,e^{-x}dx = \frac{1}{2}$

d. $\displaystyle\int_2^\infty \frac{1}{\sqrt{x}}dx$ diverges

e. $\displaystyle\int_1^\infty \frac{x}{x^2+4}dx$ diverges

f. $\displaystyle\int_{-\infty}^\infty \frac{1}{x^2+9}dx = \frac{\pi}{3}$

g. $\displaystyle\int_1^3 \frac{1}{x-1}dx$ diverges

h. $\displaystyle\int_2^4 \frac{1}{\sqrt{x-2}}dx = 2\sqrt{2}$

3. $\pi\int_1^2 \left(x^2+1\right)^2 dx = \pi\left(\frac{178}{15}\right)$

4. $2\pi\int_1^3 x\left(x^2\right)dx = 40\pi$

5. $\int_0^{\pi/2}\sqrt{1+\cos^2(x)}dx$

6. $\int_1^2 2\pi x^2\sqrt{1+4x^2}dx$

7. $y(t) = -5t - 25 + 28e^{t/5}$

8. $y(t) = \dfrac{25}{1+4e^{-t/4}}$

9.

a. $\sum_{n=1}^{\infty} \dfrac{(-2)^n}{(n+1)^n}$ converges

b. $\sum_{n=1}^{\infty} \dfrac{n^2}{3^n}$ converges

c) $\sum_{n=1}^{\infty} \dfrac{10^n}{n!}$ converges

d) $\sum_{n=1}^{\infty} \dfrac{n^2}{n^3+1}$ diverges

e. $\sum_{n=2}^{\infty} \dfrac{1}{n\ln^2(n)}$ converges

10.

a. $\sum_{n=1}^{\infty} (-1)^{n-1} \dfrac{1}{\sqrt{n}}$ converges conditionally

b. $\sum_{n=1}^{\infty} (-1)^{n} \dfrac{1}{n^{3/2}}$ converges absolutely

c. $\sum_{n=2}^{\infty} (-1)^{n} \dfrac{1}{n\ln(n)}$ converges conditionally

d. $\sum_{n=2}^{\infty} e^{-n}\sin(10n)$ converges absolutely

11. The radius of converges is 4. The open interval of convergence is $(-1, 7)$.

12.

$$\cos\left(x^3\right) = 1 - \frac{1}{2}x^6 + \frac{1}{4!}x^{12} - \frac{1}{6!}x^{18} + \cdots$$

13.

$$\arctan(x) = \int_0^x \frac{1}{1+t^2}dt = \int_0^x \left(1 - t^2 + t^4 - t^6 + \cdots\right)dt = x - \frac{1}{3}x^3 + \frac{1}{5}x^5 - \frac{1}{7}x^7 + \cdots$$

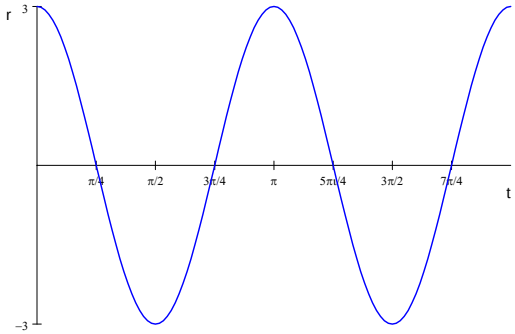
14.

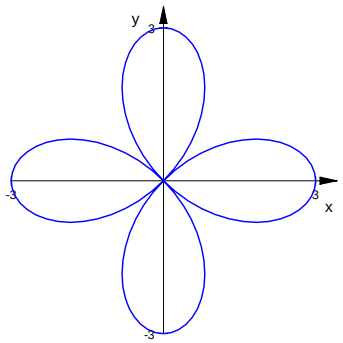
$$F(x) = \int_0^x e^{-t^2}dt = \int_0^x \left(1 - t^2 + \frac{1}{2}t^4 - \frac{1}{6}t^6 + \cdots\right)dt = x - \frac{1}{3}x^3 + \frac{1}{10}x^5 - \frac{1}{42}x^7 + \cdots$$

15.

$$\begin{aligned} &f(2) + f'(2)(x-2) + \frac{1}{2}f''(2)(x-2)^2 + \frac{1}{3!}f^{(3)}(2)(x-2)^3 \\ &= 1 + \frac{1}{2}(x-2) + \frac{3}{8}(x-2)^2 + \frac{5}{16}(x-2)^3 \end{aligned}$$

9.
i)





ii)