**Last Name:**
**First Name:**
**Instructor:**

**Math 151**
**Group Final (Fall 2003)**

This is the part of the Math 151 Final Exam that is common to all sections.
**You are not allowed to use notes, books or calculators.**
**You have exactly one hour. You will not be handed the second part of the exam before 9 AM.** If you finish this part before 9 AM, hand in your paper to the proctor and remain in your seat.

**Points**

**1._____/10**

**2._____/10**

**3._____/15**

**4._____/10**

**5._____/10**

**6._____/10**

**7._____/10**

**8._____/10**

**9._____/15**

**1** (10 pts.) Determine $\int \frac{\ln(x)}{\sqrt{x}} dx$

**2** (10 pts.) Determine $\int x e^{-x/2} dx$

**3** (15 pts.) Determine $\int \frac{4x^2 + 7x - 3}{(x+1)^2 (x-2)} dx$

**4** (10 pts.) Determine whether the improper integral $\int_{2\sqrt{3}}^{\infty} \frac{1}{x^2 + 4} dx$ converges or diverges, and its value in the case of convergence.

**5** (10 pts.) Determine whether the infinite series $\sum_{n=1}^{\infty} \frac{2^n n^2}{n!}$ converges or diverges.

**6** (10 pts.) Determine whether the infinite series $\sum_{n=1}^{\infty} \frac{(-1)^n n}{n^3+1}$ converges absolutely, converges conditionally or diverges.

**7** (10 pts.) Determine the radius of convergence and the open interval of convergence of the power series $\sum_{n=1}^{\infty} \frac{(x+4)^n}{n^2}$. (You need not investigate the series at the endpoints of the interval.)

**8** (10 pts.) Given that $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$, determine the Maclaurin series for $F(x) = \int_0^x \frac{e^t - 1 - t}{t^2} dt$. (Display the first 4 terms and the term involving $x^n$. You may leave your answer in terms of the factorial.)

**9** Let

$$r = f(\theta) = 1 + 2\cos(\theta).$$

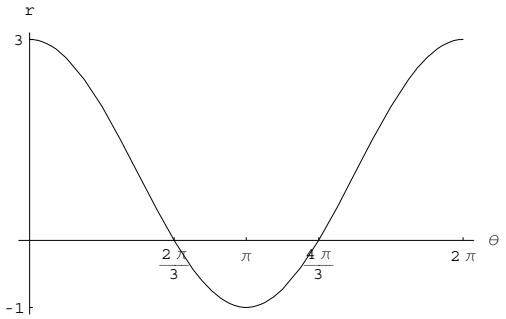
a) (5 pts) Sketch the graph of $r = f(\theta)$ in the Cartesian $\theta r$-plane on the interval $[0, 2\pi]$. Indicate the values of $\theta$ at which $f(\theta) = 0$ and the points at which $f$ attains a maximum or minimum value.

b) (10 pts.) Sketch the graph of $r = f(\theta)$ as a polar equation in the $xy$-plane (ie, $x = r\cos(\theta)$, $y = r\sin(\theta)$).

**Answers**

1. $2\sqrt{x}\ln(x) - 4\sqrt{x}$
2. $-2xe^{-x/2} - 4e^{-x/2}$
3. $\dfrac{-2}{x+1} + \ln(|x+1|) + 3\ln(|x-2|)$
4. converges to $\pi/12$
5. converges
6. converges absolutely
7. radius of convergence is 1; open interval of convergence is $(-5,-3)$
8. $F(x) = \dfrac{1}{2}x + \dfrac{1}{2\cdot 3!}x^2 + \dfrac{1}{3\cdot 4!}x^3 + \dfrac{1}{4\cdot 5!}x^4 + \cdots + \dfrac{1}{n(n+1)!}x^n + \cdots$
9.

a)




b)