Last Name:
Name:
Instructor:

Math 151
Group Final (Spring 2002)

You are not allowed to use notes, books or calculators.
You have two hours.
Simplify your responses as much as possible.

Points

1.

2.

3.

4.

5.

6.

7.

8.

9.

10.

11.

In problems $1-3$, compute the required indefinite integral.

1 (8 pts.)

$$\int x\cos(5x)\,dx$$

2 (7 pts.)

$$\int \cos^2(x)\sin^3(x)\,dx$$

3 (8 pts.)

$$\int \frac{x-19}{(x-3)\,(x+1)}dx$$

4 (10 pts.) Determine whether the improper integral

$$\int_{-\infty}^{0} xe^{x} dx$$

converges or diverges, and its value in case it converges.

5. Consider the improper integral

$$\int_0^2 \frac{x}{x^2-4}dx$$

a) (3 pts.) Why is the integral an improper integral?

b) (8 pts.) Determine whether the improper integral converges or diverges, and its value in case it converges.

6 (7 pts.) Use the ratio test to determine whether the infinite series

$$\sum_{n=1}^{\infty} (-1)^n \frac{n!}{10^n}$$

converges absolutely or diverges.

7 (10 pts.) Determine whether the infinite series

$$\sum_{n=1}^{\infty} \frac{n^2}{4n^4 - 10n + 2}$$

converges or diverges.

Hint: Use the limit comparison test.

8 (8 pts.) Determine whether the infinite series

$$\sum_{n=2}^{\infty} (-1)^n \frac{1}{e^n}$$

converges absolutely, converges conditionally or diverges. Justify your response.

9 (8 pts.) Determine the radius of convergence and the open interval of convergence of the power series

$$\sum_{n=1}^{\infty} (-1)^{n-1} \frac{n^2}{4^n} (x-2)^n$$

(You need not worry about the endpoints of the interval).

10. We have

$$\arctan(x) = x - \frac{1}{3}x^3 + \frac{1}{5}x^5 - \frac{1}{7}x^7 + \cdots$$

($\arctan(x) = \tan^{-1}(x)$).

a) (5 pts.) Determine

$$\lim_{x \to 0} \frac{\arctan(x^2) - x^2}{x^6}$$

by using the Maclaurin series of $\arctan(x)$ (**Do not use L'Hôpital's rule)**

b) (5 pts.) Let

$$F(x) = \int_0^x \frac{\arctan(t^2) - t^2}{t^6} dt.$$

Determine the first three terms of the Maclaurin series for F.

11. Let $f(\theta) = 2 + 4\cos(\theta)$.

a) (5 pts.) Sketch the graph of $r = f(\theta)$ in the Cartesian θr-plane on the interval $[0, 2\pi]$. Indicate the values of θ at which f $f(\theta) = 0$.

b) (8 pts.) Sketch the graph of $r = f(\theta)$ in the Cartesian xy-plane if r and θ are polar coordinates, i.e., $x = r\cos(\theta)$ and $y = r\sin(\theta)$.

Answers

1.

$$\int x\cos(5x)\,dx = \frac{1}{5}x\sin(5x) + \frac{1}{25}\cos(5x) + C.$$

2.

$$\int \cos^2(x)\left(1-\cos^2(x)\right)\sin(x)\,dx = -\frac{1}{3}\cos^3(x) + \frac{1}{5}\cos^5(x) + C.$$

3.

$$\int \frac{x-19}{(x-3)(x+1)}dx = -4\ln(|x-3|) + 5\ln(|x+1|) + C.$$

4.

$$\int_{-\infty}^{0} xe^x dx = -1.$$

5.
a) The integral is improper since

$$\lim_{x\to 2-}\frac{x}{x^2-4} = -\infty.$$

b) The improper integral diverges.

6. The series diverges.

7. The series also converges.

8. The series converges absolutely.

9. The radius of convergence is 4 and the open interval of convergence is $(-2,6)$.

10.
a)

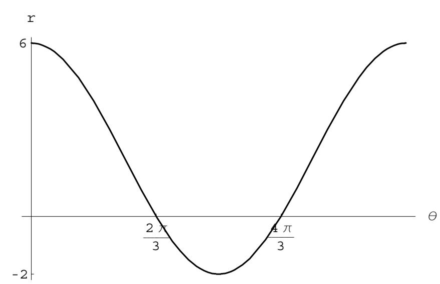
$$\lim_{x\to 0}\frac{\arctan(x^2)-x^2}{x^6} = \lim_{x\to 0}\frac{-\frac{1}{3}x^6+\frac{1}{5}x^{10}-\frac{1}{7}x^{14}+\cdots}{x^6} = \lim_{x\to 0}\left(-\frac{1}{3}+\frac{1}{5}x^4-\frac{1}{7}x^8+\cdots\right) = -\frac{1}{3}$$

b)

$$F(x) = \int_0^x \frac{\arctan(t^2) - t^2}{t^6} dt = \int_0^x \left(-\frac{1}{3} + \frac{1}{5}t^4 - \frac{1}{7}t^8 + \cdots \right) dt = -\frac{1}{3}x + \frac{1}{25}x^5 - \frac{1}{63}x^9 + \cdots$$

11.

a)



b)