Last Name:
Name:
Instructor:

Math 151
Group Final (Spring 2003)

This is the part of the Math 151 Final Exam that is common to all sections.

You are not allowed to use notes, books or calculators.

You have exactly one hour. You will not be handed the second part of the exam before 9 AM. If you finish this part before 9 AM, hand in your paper to the proctor and remain in your seat.

Points

1.

2.

3.

4.

5.

6.

7.

8.

9.

1 (10 pts.) Determine

$$\int xe^{-4x}dx$$

2 (15 pts.) Determine

$$\int x^2 \ln(x)\, dx$$

3 (10 pts.) Determine

$$\int \frac{x-23}{x^2+3x-10}dx$$

4 (10 pts.) Evaluate

$$\int_0^\infty 4^{-x}dx$$

5 (10 pts.) Use the integral test to determine whether the infinite series

$$\sum_{n=1}^{\infty} \frac{n^3}{n^4 + 4}$$

converges or diverges (you need not justify the applicability of the test).

6 (10 pts.) Determine whether

$$\sum_{n=1}^{\infty} (-1)^n \frac{2^n}{n!}$$

converges absolutely or conditionally.

7 (10 pts.) Determine the radius of convergence and the open interval of convergence of the power series

$$\sum_{n=0}^{\infty} (-1)^n \frac{n}{3^n} (x-4)^n$$

(you need not investigate the series at the endpoints of the interval)

8 (10 pts.) Let

$$F(x) = \int_0^x \frac{1}{1-t^2} dt.$$

Determine the Maclaurin series for F (display the first 3 terms and the term involving x^{2n+1}).

9. Let

$$r = f(\theta) = 2 - \cos(\theta).$$

a) (5 pts) Sketch the graph of $r = f(\theta)$ the Cartesian θr-plane on the interval $[0, 2\pi]$.

b) (10 pts.) Sketch the graph of $r = f(\theta)$ in the Cartesian xy-plane if r and θ are polar coordinates ($x = r\cos(\theta)$, $y = r\sin(\theta)$).

Solutions

1. Set $u = x$ and $dv = e^{-4x}dx$, so that

$$du = dx \text{ and } v = \int e^{-4x}dx = -\frac{1}{4}e^{-4x}.$$

Therefore,

$$\begin{aligned}\int xe^{-4x}dx &= \int udv \\ &= uv - \int vdu \\ &= x\left(-\frac{1}{4}e^{-4x}\right) + \frac{1}{4}\int e^{-4x}dx \\ &= -\frac{1}{4}xe^{-4x} - \frac{1}{16}e^{-4x} + C.\end{aligned}$$

2. Set $u = \ln(x)$ and $dv = x^2dx$, so that

$$du = \frac{1}{x}dx \text{ and } v = \int x^2dx = \frac{1}{3}x^3.$$

Therefore,

$$\begin{aligned}\int \ln(x)\, x^2dx &= \int udv \\ &= uv - \int vdu \\ &= \ln(x)\left(\frac{1}{3}x^3\right) - \frac{1}{3}\int x^3\left(\frac{1}{x}\right)dx \\ &= \frac{1}{3}x^3\ln(x) - \frac{1}{3}\int x^2dx \\ &= \frac{1}{3}x^3\ln(x) - \frac{1}{9}x^3 + C.\end{aligned}$$

3. We have $x^2 + 3x - 10 = (x+5)(x-2)$.

$$\frac{x-23}{(x+5)(x-2)} = \frac{A}{x+5} + \frac{B}{x-2}$$
$$\Leftrightarrow x - 23 = A(x-2) + B(x+5)$$

Set $x = -5$:

$$-28 = -7A \Rightarrow A = 4.$$

Set $x = 2$:

$$-21 = 7B \Rightarrow B = -3.$$

Therefore,

$$\frac{x-23}{x^2+3x-10} = \frac{4}{x+5} - \frac{3}{x-2}.$$

Thus,

$$\int \frac{x-23}{x^2+3x-10}dx = 4\int \frac{1}{x+5}dx - 3\int \frac{1}{x-2}dx$$
$$= 4\ln(|x+5|) - 3\ln(|x-2|) + C$$

4.

$$\int_0^b 4^{-x}dx = -\frac{1}{\ln(4)}4^{-x}\Big|_0^b = -\frac{1}{\ln(4)}4^{-b} + \frac{1}{\ln(4)}$$
$$\lim_{b\to+\infty}\int_0^b 4^{-x}dx = \frac{1}{\ln(4)}$$

5.

$$\int_1^b \frac{x^3}{x^4+4}dx = \frac{1}{4}\ln\left(x^4+4\right)\Big|_1^b$$
$$= \frac{1}{4}\ln\left(b^4+4\right) - \frac{1}{4}\ln(5).$$

Therefore,

$$\lim_{b\to\infty}\int_1^b \frac{x^3}{x^4+4}dx = +\infty.$$

Therefore, the series diverges.

6.

$$\lim_{n\to\infty}\frac{\frac{2^{n+1}}{(n+1)!}}{\frac{2^n}{n!}} = \lim_{n\to\infty}\frac{2}{n+1} = 0 < 1.$$

Therefore the series converges absolutely.

7.

$$\lim_{n\to\infty}\frac{\frac{n+1}{3^{n+1}}\left|x-4\right|^{n+1}}{\frac{n}{3^n}\left|x-4\right|^{n}}=\left|x-4\right|\lim_{n\to\infty}\left(\frac{n+1}{3n}\right)=\frac{1}{3}\left|x-4\right|$$

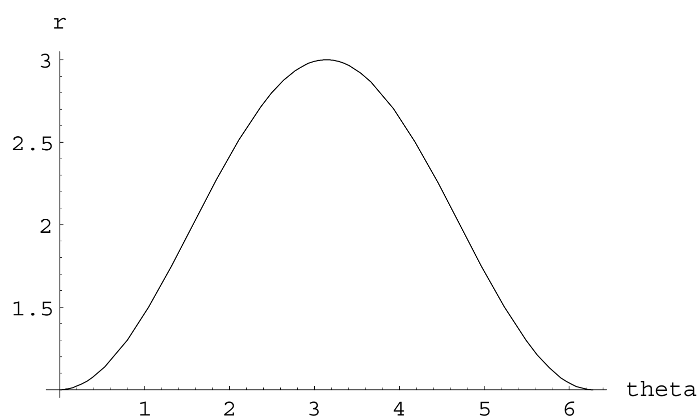
Therefore, the radius of convergence is 3. The open interval of convergence is $(1,7)$.

8.

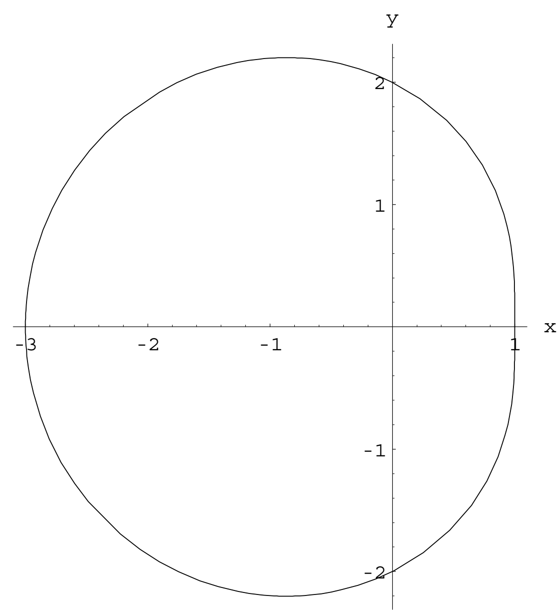
$$\begin{aligned}F\left(x\right)&=\int_0^x\frac{1}{1-t^2}dt\\&=\int_0^x\left(1+t^2+t^4+\cdots+t^{2n}+\cdots\right)dt\\&=x+\frac{1}{3}x^3+\frac{1}{5}x^5+\cdots\frac{1}{2n+1}x^{2n+1}+\cdots\end{aligned}$$

9.

a)



b)


y
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1
x
-3
-2
-1
1
-1
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