Last Name:
First Name:
Instructor:

Math 151
Group Final (Spring 2004)

This is the part of the Math 151 Final Exam that is common to all sections.
You are not allowed to use notes, books, calculators, personal stereos or cell phones.
You have exactly one hour. You will not be handed the second part of the exam before 9 AM. If you finish this part before 9 AM, hand in your paper to the proctor and remain in your seat.
Circle all answers.

Points

1._____/10

2._____/10

3._____/15

4._____/5

5._____/10

6._____/10

7._____/10

8._____/10

9._____/10

10._____/10

1. (10 pts.) Determine

$$\int x \sin(x/3)\, dx$$

2. (10 pts.) Determine

$$\int \sin^2(4x)\, dx$$

3. (a) (10 pts.) Determine

$$\int \frac{\ln(x)}{x^{1/3}} dx$$

(b) (5 pts) Determine whether the improper integral

$$\int_0^1 \frac{\ln(x)}{x^{1/3}} dx$$

converges or diverges and its value in the case of convergence.

4. (5 pts) Find the *form* of the partial fraction decomposition for the expression

$$\frac{3x^4 + 13x^3 + 7x^2 - 17x + 14}{(x+2)(x^2+1)(x+3)^2}$$

You do not need to solve for the coefficients.

5. (10 pts.) Find the antiderivative:

$$\int \frac{7x + 11}{(x+3)(x-2)} dx$$

6. (10 pts.) Use the integral test to determine if the infinite series

$$\sum_{n=1}^{\infty} n e^{-n^2}$$

converges or diverges. (You need not justify the applicability of the test.)

7. (10 pts.) Determine whether the infinite series

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{2n^3+3}$$

converges absolutely, converges conditionally, or diverges.

8. (10 pts.) Determine the radius of convergence and the open interval of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{(x-1)^n}{2^n n^{3/2}}.$$

(You need not investigate the series at the endpoints of the interval.)

9. (10 pts.) Given that

$$\arctan(x) = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots, -1 \leq x \leq 1$$

determine the first four nonzero terms of the Maclaurin series for

$$\frac{d}{dx} \arctan\left(x^2\right)$$

Simplify as much as possible.

10. Let

$$r = f(\theta) = 3\sin(2\theta).$$

(a) (5 pts) Sketch the graph of $r = f(\theta)$ in the Cartesian θr-plane on the interval $[0, 2\pi]$. Indicate the values of θ at which $f(\theta) = 0$ and the points at which f attains a maximum or minimum value.

(b) (5 pts.) Sketch the graph of $r = f(\theta)$ as a polar equation in the xy-plane (ie, $x = r\cos(\theta)$, $y = r\sin(\theta)$).

Answers

1.

$$\int x \sin(x/3)\, dx = 9 \sin(x/3) - 3x \cos(x/3)$$

2.

$$\int \sin^2(4x)\, dx = -\frac{1}{8} \cos(4x) \sin(4x) + \frac{1}{2} x$$

3.

a)

$$\int \frac{\ln(x)}{x^{1/3}} dx = \frac{3}{2} x^{2/3} \ln(x) - \frac{9}{4} x^{2/3}$$

b)

$$\int_0^1 \frac{\ln(x)}{x^{1/3}} dx = -\frac{9}{4}$$

4.

$$\frac{3x^4 + 13x^3 + 7x^2 - 17x + 14}{(x+2)(x^2+1)(x+3)^2} = \frac{A}{x+2} + \frac{Bx+C}{x^2+1} + \frac{D}{(x+3)} + \frac{E}{(x+3)^2}$$

5.

$$\int \frac{7x+11}{(x+3)(x-2)} dx = 2 \ln|x+3| + 5 \ln|x-2|$$

6. $\sum_{n=1}^{\infty} n e^{-n^2}$ converges since

$$\int_1^{\infty} x e^{-x^2} dx$$

converges (and has the value $e^{-1/2}$: You need to confirm this).

7.

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{2n^3 + 3}$$

converges absolutely.

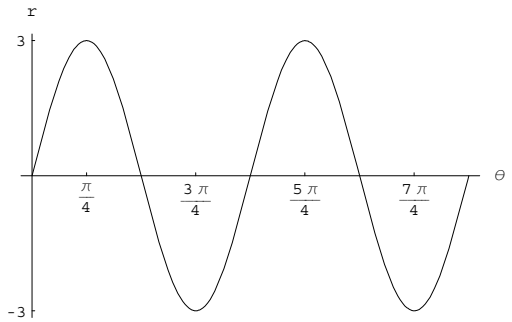
8. The radius of convergence is 2 and the open interval of convergence is $(-1, 3)$.

9.

$$\frac{d}{dx} \arctan\left(x^2\right) = 2x - 2x^5 + 2x^9 - 2x^{13} + \cdots$$

10.

a)



b)