Math 150
Group Final (Fall 2007)
Solutions

1.

$$\lim_{x\to 2-}\frac{x^2+x-6}{(x-2)^2}=\lim_{x\to 2-}\frac{(x+3)(x-2)}{(x-2)^2}=\lim_{x\to 2-}\frac{x+3}{(x-2)}=-\infty,$$

since $\lim_{x\to 2}(x+3)=5>0,\ \ x-2<0$ if $x<2$ and $\lim_{x\to 2}=0$.

2.

$$\lim_{x\to+\infty}x2^{-x}=\lim_{x\to+\infty}\frac{x}{2^x}=\lim_{x\to+\infty}\frac{1}{\ln(2)\,2^x}=0.$$

3.

$$\lim_{x\to\pi/6}\frac{2\sin(x)-1}{6x-\pi}=\lim_{x\to\pi/6}\frac{2\cos(x)}{6}=\frac{1}{3}\cos(\pi/6)=\frac{1}{3}\left(\frac{\sqrt{3}}{2}\right)=\frac{\sqrt{3}}{6}.$$

4.

$$\frac{d}{dx}\left(x^2\cos\left(\frac{x}{2}\right)\right)=2x\cos\left(\frac{x}{2}\right)-\frac{1}{2}x^2\sin\left(\frac{x}{2}\right).$$

5.

$$\begin{aligned}\frac{d}{dx}\ln\left((x+1)^{1/3}(x+2)\right)&=\frac{d}{dx}\left(\frac{1}{3}\ln(x+1)+\ln(x+2)\right)\\&=\frac{1}{3(x+1)}+\frac{1}{x+2}.\end{aligned}$$

6.

$$\frac{d}{dx}\arcsin\left(x^2\right)=\frac{1}{\sqrt{1-x^4}}(2x)=\frac{2x}{\sqrt{1-x^4}}$$

7.

$$\begin{aligned}\frac{d}{dx}\left(\frac{x^2+1}{x^3+x}\right)&=\frac{2x\left(x^3+x\right)-\left(x^2+1\right)\left(3x^2+1\right)}{\left(x^3+x\right)^2}\\&=\frac{2x^4+2x^2-3x^4-x^2-3x^2-1}{\left(x^3+x\right)^2}\\&=\frac{-x^4-2x^2-1}{\left(x^3+x\right)^2}=-\frac{\left(x^2+1\right)^2}{x^2\left(x^2+1\right)^2}=-\frac{1}{x^2}\end{aligned}$$

8.
a) We have

$$f'(x)=\frac{d}{dx}x^{3/4}=\frac{3}{4}x^{-1/4}=\frac{3}{4x^{1/4}}$$

Therefore,

$$f'(16)=\frac{3}{4\left(16^{1/4}\right)}=\frac{3}{8}.$$

Thus,

$$L_{16}(x)=f(16)+f'(16)(x-16)=8+\frac{3}{8}(x-16).$$

b)

$$\begin{aligned}(16.2)^{3/4} = f(16.2) &\cong L_{16}(16.02) \\ &= 8 + \frac{3}{8}(0.2) = 8.075\end{aligned}$$

9. Let θ be the angle between the ground level and the observer's line sight, and let y be the height of the rocket. Thus,

$$\tan(\theta) = \frac{y}{4}.$$

Therefore,

$$\frac{1}{\cos^2(\theta)}\frac{d\theta}{dt} = \frac{1}{4}\frac{dy}{dt} = \left(\frac{1}{4}\right)(400) = 100.$$

Thus,

$$\frac{d\theta}{dt} = 100\cos^2(\theta).$$

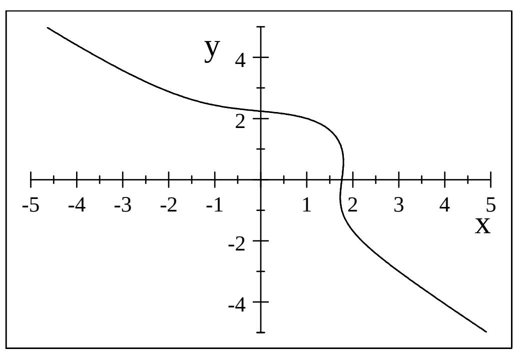
At the instant $y = 8$,

$$\cos(\theta) = \frac{4}{\sqrt{8^2+4^2}} = \frac{4}{\sqrt{80}} = \frac{4}{4\sqrt{5}} = \frac{1}{\sqrt{5}}$$

Therefore,

$$\frac{d\theta}{dt} = 100\cos^2(\theta) = 100\left(\frac{1}{5}\right) = 20$$

radians/min.
10.



$$3x^2 + 3y^2\frac{dy}{dx} - \frac{dy}{dx} + 2 = 0$$

$\Rightarrow$

$$\left(3y^2-1\right)\frac{dy}{dx} = -3x^2 - 2 \Rightarrow \frac{dy}{dx} = \frac{-3x^2-2}{3y^2-1}.$$

Since $y(1) = 2$,

$$y'(1) = \left.\frac{-3x^2-2}{3y^2-1}\right|_{x=1,\ y=2} = \frac{-3-2}{12-1} = -\frac{5}{11}.$$

11. We have

$$f'(x) = \frac{d}{dx}\left(-\frac{1}{12}x^4 - \frac{1}{3}x^3 + 4x^2\right) = -\frac{1}{3}x^3 - x^2 + 8x$$

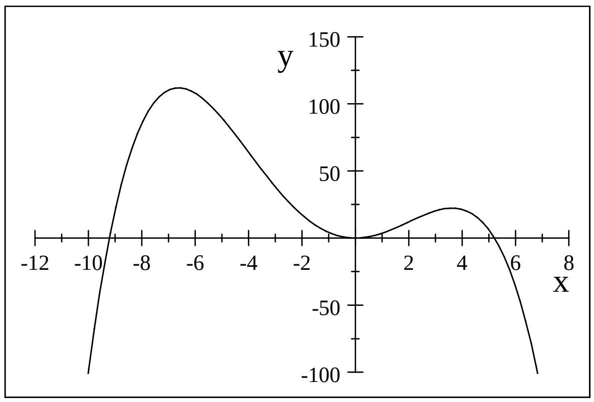
and

$$f''(x) = \frac{d}{dx}\left(-\frac{1}{3}x^3 - x^2 + 8x\right) = -x^2 - 2x + 8 = -(x+4)(x-2)$$

Therefore,

$$f''(x) < 0 \text{ if } x < -4,\ f'(x) > 0 \text{ if } -4 < x < 2 \text{ and } f''(x) < 0 \text{ if } x > 2.$$

Thus, the graph of f is concave down on $(-\infty, -4]$, concave up on $[-2, 2]$ and concave down on $[2, +\infty)$. The x-coordinates of the points of inflection of the graph of f are -4 and 2.



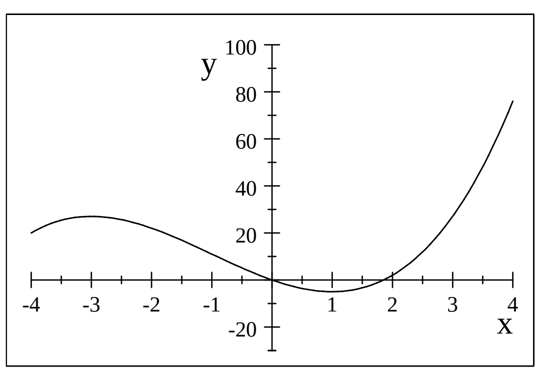
12. We have

$$f'(x) = \frac{d}{dx}\left(x^3 + 3x^2 - 9x\right) = 3x^2 + 6x - 9 = 3(x+3)(x-1).$$

Thus, the critical points of f in $[-4, 4]$ are -3 and 1. We have

$$f(-3) = 27,\ f(1) = -5,\ f(-4) = 20,\ f(4) = 76.$$

Therefore, the absolute maximum of f on $[-4, 4]$ is $f(4) = 76$, and the absolute minimum of f on $[-4, 4]$ is $f(1) = -5$.



13.
a) We have

$$f'(x) = -\frac{2x-1}{(x+2)^2(x-3)^2} \text{ if } x \neq -2 \text{ and } x \neq 3.$$

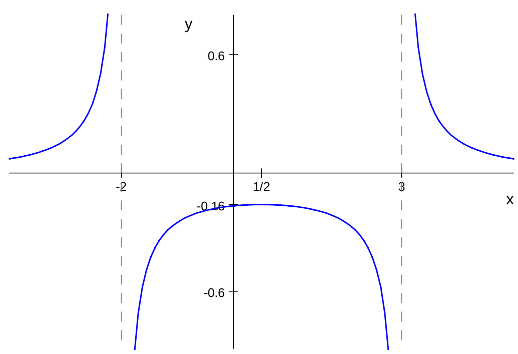
Therefore, $f'(x) = 0 \Leftrightarrow x = 1/2$.

x		-2		$\frac{1}{2}$		3	
$f'(x)$	$+$	undef.	$+$	0	$-$	undef.	$-$
f	incr.	undef.	incr..	loc. max.	decr.	undef.	decr.

The function is increasing on $(-\infty, -2)$, increasing on $(-2, 1/2]$, decreasing on $[1/2, 3)$ and decreasing on $(3, +\infty)$. Thus, f has a local maximum at $1/2$.
b) The vertical asymptotes are $x = -2$ and $x = 3$. Since

$$\lim_{x\to\pm} f(x) = 0,$$

the x-axis is the horizontal asyptoteat $\pm\infty$.



14.

$$\int_1^2 \frac{d}{dx}\left(\frac{1}{\sqrt{9-x^2}}\right) dx = \left.\frac{1}{\sqrt{9-x^2}}\right|_{x=1}^{x=2} = \frac{1}{\sqrt{5}} - \frac{1}{\sqrt{8}}$$

15.

$$\frac{d}{dx}\int_{\pi/4}^{x} \cos\left(t^2\right) dt = \cos\left(x^2\right)$$

16. We set $u = 1 + x^3$ so that $du = 3x^2 dx$. Thus,

$$\int \frac{x^2}{\left(1+x^3\right)^{1/3}} dx = \frac{1}{3}\int \frac{1}{u^{1/3}} du = \frac{1}{3}\int u^{-1/3} du = \frac{1}{3}\left(\frac{3}{2}u^{2/3}\right) = \frac{1}{2}\left(1+x^3\right)^{2/3}.$$

17. We set $u = \ln(x)$ so that $du = (1/x)\,dx$. Therefore

$$\int_{e^2}^{e^4} \frac{\ln(x)}{x} dx = \int_{\ln(e^2)}^{\ln(e^4)} u du = \int_2^4 u du = \left.\frac{1}{2}u^2\right|_2^4 = 8 - 2 = 6$$

18. We set $u = 4x$ so that $du = 4dx$.

$$\int \frac{1}{1+16x^2}dx = \frac{1}{4}\int \frac{1}{1+u^2}du = \frac{1}{4}\arctan(u) = \frac{1}{4}\arctan(4x) + C$$

19.The position of the object at $t = 8\pi$ is

$$\begin{aligned} 4 + \int_0^{2\pi} \sin\left(\frac{t}{4}\right) dt &= 4 + 4\int_{u=0}^{u=\pi/2} \sin(u)\, du \\ &= 4 + 4\left(-\cos(u)\big|_0^{\pi/2}\right) \\ &= 4 + 4(0+1) = 8.. \end{aligned}$$