

Last Name:
First Name:
Instructor:

Math 151
Group Final (Fall 2007)

You are not allowed to use notes, books, calculators, personal stereos or cell phones.

You have exactly two hours.

Write clearly so that you can avoid mistakes and count on partial credits. Carry out the obvious simplifications so that you can display your answers in an easily readable manner.

The following list is for the recording of the points only. Do not write your answers on this page.

Points

- 1 _____/8
- 2 _____/6
- 3 _____/6
- 4 _____/8
- 5 _____/8
- 6 _____/8
- 7 _____/8
- 8 _____/8
- 9 _____/10
- 10 _____/6
- 11 _____/8
- 12 _____/8
- 13 _____/8

Total: /100

1 (8 pts.) Determine

$$\int \arctan\left(\frac{x}{4}\right) dx$$

2 (6 pts.) Determine

$$\int \frac{x - 17}{x^2 + x - 12} dx$$

3 (6 pts.) Determine

$$\int x \cos(\pi x) dx.$$

4.

a) (2 pts.) Why is

$$\int_3^4 \frac{1}{(x-3)^{2/3}} dx$$

an improper integral?

b) (6 pts.) Determine whether the integral of part a) converge or diverges, and its value in case it converges.

5 (8 pts.) Use a comparison test to determine whether the improper integral

$$\int_1^{\infty} \frac{1}{\sqrt{x^4 + 1}} dx$$

converges or diverges. You need not determine the value of the improper integral in case of convergence.

6 (8 pts.) Determine the length of the graph of

$$y = \cosh(x)$$

on the interval $[0, 1]$. Express the result in terms of the natural exponential function (Hint: Make use of the identity that links $\cosh^2(x)$ to $\sinh^2(x)$).

7 (8 pts.) Determine the solution of the initial value problem

$$3t \frac{dy(t)}{dt} = y(t) - t, \quad y(1) = 4$$

(assume that $t > 0$).

8 (8 pts.) Determine the solution of the initial value problem

$$\frac{dy(x)}{dx} = y^2(x) + 1, \quad y\left(\frac{\pi}{4}\right) = 0$$

9. Let

$$r = f(\theta) = 1 - 2\sin(\theta).$$

a) (4 pts.) Sketch the graph of $r = f(\theta)$ in the Cartesian θr -plane on the interval $[0, 2\pi]$. Indicate the values of θ at which $f(\theta) = 0$ and the points at which f attains a local maximum or minimum value.

b) (6 pts.) Sketch the graph of $r = f(\theta)$, where $0 \leq \theta \leq 2\pi$, as a polar equation in the xy -plane (i.e., $x = r \cos(\theta)$, $y = r \sin(\theta)$).

10 (6 pts.) Determine whether the infinite series

$$\sum_{n=1}^{\infty} \frac{2^n}{n^{10}}$$

converges or diverges.

11 (8 pts.) Determine whether the infinite series

$$\sum_{n=1}^{\infty} (-1)^{n-1} \frac{e^{1/n}}{n^2 + 1}$$

converges absolutely, converges conditionally or diverges (you may have to make use of a comparison test)

12 (8 pts.) Determine the radius of convergence and the open interval of convergence of the power series

$$\sum_{n=1}^{\infty} (-1)^{n-1} \frac{1}{n} (x-1)^n$$

(You need not investigate the series at the endpoints of the interval).

13 (8 pts.) Let $f(x) = x^{2/3}$. Determine the first four terms of the Taylor series for f in powers of $x-1$.